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PID Stabilization of a Position-Controlled Robot Manipulator Acting Independently or in Collaboration with Human Arm

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Abstract

In this paper we develop framework for PID stabilization of a robot manipulator when using an object independently or in collaboration with a human arm. In both applications, the manipulator is equipped with a wrist sensor represented by an impedance. A second order manipulator transfer function along each coordinate direction is assumed. The aim of the paper is to design a PID controller when measurement of contact force, available via wrist sensor, is used to command the position-controlled manipulator to a desired position and/or force profile. Necessary and sufficient conditions for stability of the closed-loop system are developed using Hermite-Biehler Theorems. The theorems have been used to analyze stability of polynomials defined over the set of real numbers. An algorithm for synthesis of PID controllers using linear matrix inequalities is developed. The theoretical framework presented in this paper can be easily adapted to other low order manipulator transfer functions.

Introduction

Robot manipulators are used in a number of industrial and service applications (Hunt, 1983; Synder, 1985; Engelberger, 1989; Tao et al., 1990; Luh and Zheng, 1987; Zheng and Luh, 1989; Al-Jarrah and Zheng, 1996; Al-Jarrah and Zheng, 1997). In most cases, where the objective is to manipulate an inertial object, a robot can be considered as a positioning device that decouples the motion along each coordinate direction, and can be approximated with a second order transfer function (Xu and Paul, 1988; Al-Jarrah and Zheng, 1996; Kazerooni, 1990). Also, in position and/or force control applications, a high impedance wrist-sensor is normally used as end-effector, which can be effectively modeled with impedance. The wrist-sensor stiffness is usually high, on the order of 10^5 oz-in.

The arm-manipulator coordination problem visualizes a human arm and a robot manipulator jointly handling an inertial object in unstructured workspace. A human operator can acquire visual knowledge of the environment relatively quickly, possess the necessary intelligence to analyze the situation, and take quick and effective decisions. In the arm-manipulator coordination scheme, the intelligence of the arm helps perform complex functions (e.g., task planning, obstacle avoidance, etc.) while the manipulator performs the load sharing function. The arm-manipulator coordination for load sharing has been discussed by researchers (Al-Jarrah and Zheng, 1996; Al-Jarrah and Zheng, 1997; Kazerooni, 1990; Iqbal and Zheng, 1997; Iqbal and Zheng, 1999; Ikeura and Inooka, 1995; Ikeura and Mizutani, 1998; Rahman et al.,

1999) and several solutions to the problem, (e.g., compliant motion control, reflexive motion control, and model predictive control) have been proposed. For example, the manipulator was required to possess negative stiffness to ensure compliance in load sharing tasks (Al-Jarrah and Zheng, 1996). In the reflexive motion control scheme (Al-Jarrah and Zheng, 1997), a supervisory loop was added that acted like a force control to the compliant controller to compensate for the sluggishness of the robot. The controller thus anticipated the arm movement, and applied timely corrections to improve the manipulator response. Model predictive control strategies have also been proposed to solve the arm-manipulator coordination problem (Iqbal and Zheng, 1997; Iqbal and Zheng, 1999). In predictive control schemes, the observed manipulator output was used in an optimizing controller to command the manipulator such that the predicted arm force went to zero.

In this paper we study the stability and controller design of the closed-loop system formed by the positioning manipulator, the wrist-sensor, and a PID controller. A secondary loop is added due to the presence of the human arm. We use the Hermite-Biehler framework (Roy and Iqbal, 2002; Datta et al., 1999; Ho et al., 2000) to analyze the stability of the closed-loop system. Hermite Biehler and generalized Hermite-Biehler Theorems characterize the stability of a given polynomial and provide information on right half-plane (RHP) root locations. We show how the characteristic polynomial of the robot-sensor and arm-manipulator plants can be cast in the Hermite-Biehler framework. The paper then discusses synthesis of the PID controller for the problem. A general analysis-synthesis

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framework is developed that can be applied to similar plant models.

Methods

Notation.--In the following, \wedge denotes logical AND, \vee denotes logical OR, $\mathbb{R} = (-\infty, \infty)$, $\mathbb{R}^+ = (0, \infty)$, $\mathbb{R}^- = (-\infty, 0)$, $\{\mathbb{R} \neq 0\} = \mathbb{R}^+ \cup \mathbb{R}^-$, L_+ denotes the set of non-negative integers, $\mathbb{R}^{m \times n}$ denotes a $m \times n$ real matrix, C denotes the set of complex numbers, and \emptyset denotes a null set.

Problem Formulation and Stability Analysis.--The arm-manipulator coordination problem can be visualized as a human arm and a robot manipulator jointly handling an inertial object (Fig. 1). The manipulator is modeled as a

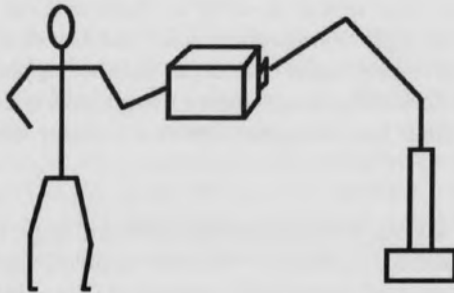


Fig. 1. The conceptual visualization of the arm-manipulation coordination problem; a human arm and a robot manipulator jointly handle an inertial object.

positioning device, which decouples the dynamics of the robot and provides position tracking in Cartesian coordinates. In particular, for the Puma 560 robot, a natural frequency of 2 Hz can be assumed (Xu and Paul, 1988; Al-Jarrah and Zheng, 1996; Kazerooni, 1990). A high-impedance wrist sensor is used to sense the environmental forces and moments.

The human arm is modeled as a black box neglecting the behavior of the musculo-skeletal system (Kazerooni, 1990). The arm possesses an impedance, which arises from the visco-elastic properties of the biological muscles. The desired arm trajectory is planned in the central nervous system, and no apriori knowledge of it is assumed. Further, it is assumed that the speed of manipulation task is small, such that the Coriolis and other nonlinear effects can be neglected. It is further assumed that the only forces acting on the object are the arm force (f_a), the manipulator force (f_m), and the force of gravity (f_g). Then the dynamics of the problem (Fig. 2) can be solved from Newton's laws of motion represented by the following equations, where in order to simplify the analysis, only a one-dimensional view of the problem is considered.

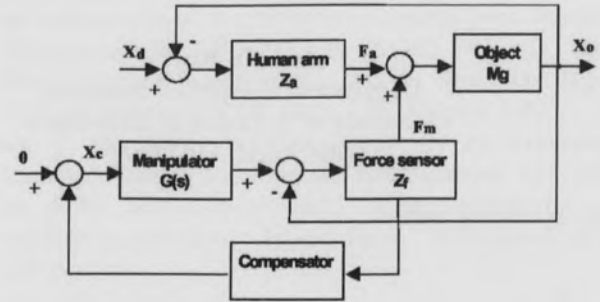


Fig. 2. Block diagram representation of the arm-manipulator coordination problem.

$$m\ddot{x}_o = f_a + f_m$$

$$f_a = k_a(x_d - x_o) + c_a(\dot{x}_d - \dot{x}_o)$$

$$f_m = k_m(Gx_c - x_o)$$

where f_a denotes arm force, f_m denotes manipulator force, k_a is arm stiffness, c_a is arm damping (assumed as viscous), k_m is manipulator (wrist-sensor) stiffness, x_d is desired object position, x_c is manipulator command, x_o is the object position, and $G(s)$ represents the manipulator transfer function given as

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The transfer functions relating manipulator command to arm-force and manipulator force are given by

$$\frac{f_a}{x_c} = -\frac{k_m(c_a s + k_a)}{ms^2 + c_a s + (k_a + k_m)} G(s) \text{ and}$$

$$\frac{f_m}{x_c} = \frac{mk_m s^2}{ms^2 + c_a s + (k_a + k_m)} G(s)$$

Finally, the position of the object being manipulated is given as

$$x_o = \frac{(c_a s + k_a)x_d + k_m x_c G(s)}{ms^2 + c_a s + (k_a + k_m)}$$

where in the steady state, the object position is given by $(k_a x_d + k_m x_c) / (k_a + k_m)$. Since $k_m \gg k_a$, the object position will be primarily determined by the manipulator command (x_c). By commanding the manipulator to eliminate the arm force, we can ensure that $x_c = x_d = x_o$ (Iqbal and Zheng, 1997; Iqbal and Zheng, 1999). Accordingly, in the arm-manipulator coordination problem (i.e., the regulation

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problem) our objective is to reduce the arm force to zero. In the absence of the human arm, when the manipulator interacts with the environment, the sensor output can be controlled using a generalized force input of the form $f_g = ms^2 x_c$, such that

$$\frac{f_m}{f_g} = \frac{k_m}{ms^2 + k_m} G(s)$$

where the above representation defines a tracking problem. In the following our effort would be to develop a general framework that can handle the above as well as other similar situations. Accordingly we consider a possibly unstable process given by the following transfer function

$$G_p(s) = \frac{n_p}{d_p}(s) = \frac{\omega_n^2(as + b)}{(ms^2 + cs + k)(s^2 + 2\xi\omega_n s + \omega_n^2)} \quad (1)$$

where the first part represents a general second order transfer function, and the second part represents a position-controlled manipulator. We assume that the process given by equation (1) is controlled through unity gain feedback by a PID controller whose transfer is given by

$$G_c(s) = \frac{n_c}{d_c}(s) = \frac{K_d s^2 + K_p s + K_i}{s} \quad (2)$$

Then, the closed-loop characteristic polynomial is given as

$$\psi(s) = sd_p(s) + (K_i + s^2 K_d)n_p(s) + sK_p n_p(s) \quad (3)$$

Define $\tilde{A} = \{a, b, k, c, m, \xi, \omega_n\}$ to be the set of all system constants, and let $\kappa = \{K_p, K_i, K_d\}$ represent the controller parameters; then the design problem is defined as, given \tilde{A} , determine

$$\tilde{\kappa} \ni \text{Re}[s_\psi] < 0 \forall s_\psi, \left\{ \begin{matrix} \psi(s_\psi) = 0 \end{matrix} \right\}$$

Hermite-Biehler Theorems.--In this section we state the Hermite-Biehler and the generalized Hermite-Biehler Theorems (Datta et al., 1999; Ho et al., 2000).

Theorem 1 (Hermite-Biehler Theorem): Let

$$\delta(s) = \sum_{i=0}^n \delta_i s^i, \delta_i \in \mathbb{R} \forall i.$$

Write $\delta(s) = \delta_e(s^2) + \delta_o(s^2)$ where $\delta_{e,o}(s^2)$ are the components of $\delta(s)$ made up of even and odd powers of s , respectively. Let ω_{ej} denote the real non-negative distinct zeros of $\delta_e(-\omega^2)$ and let ω_{ok} denote the real non-negative distinct zeros of $\delta_o(-\omega^2) \forall j, k$, both arranged in ascending order of magnitude. Then $\delta(s)$ is Hurwitz stable if and only if δ_n and δ_{n-1} are of the same sign, all the zeros of $\delta_e(-\omega^2)$, $\delta_o(-\omega^2)$ are real and distinct, and the non-negative real zeros satisfy the interlacing property given by $0 < \omega_{e1} < \omega_{o1} < \omega_{e2}$

$< \omega_{o2} < \dots$

Theorem 2: Let $\delta(s) = \sum_{i=0}^n \delta_i s^i, \delta_i \in \mathbb{R} \forall i$. Write

$\delta(s) = \delta_e(s^2) + \delta_o(s^2)$ where $\delta_{e,o}(s^2)$ are the components of $\delta(s)$ made up of even and odd powers of s , respectively. For every $\omega \in \mathbb{R}$, denote $\delta(j\omega) = p(\omega) + jq(\omega)$ where $p(\omega)$ and $q(\omega)$ are given by $p(\omega) = \delta_e(-\omega^2)$ and $q(\omega) = \delta_o(-\omega^2)$; let ω_{ej} denote the real non-negative distinct zeros of $\delta_e(-\omega^2)$, and let ω_{ok} denote the real non-negative distinct zeros of $\delta_o(-\omega^2) \forall j, k$, both arranged in ascending order of magnitude. Then the following conditions are equivalent:

(1) $\delta(s)$ is Hurwitz stable,

(2) δ_n and δ_{n-1} are of the same sign, and

$$n = \text{sgn}[\delta_o] \left[\text{sgn}[p(0)] + 2 \sum_{i=1}^m (-1)^i \text{sgn}[p(\omega_{oi})] \right], n = 2m \forall m \in I_+, \omega_{om} = \infty$$

$$n = \text{sgn}[\delta_o] \left[\text{sgn}[p(0)] + 2 \sum_{i=1}^m (-1)^i \text{sgn}[p(\omega_{oi})] \right], n = 2m + 1 \forall m \in I_+$$

(3) δ_n and δ_{n-1} are of the same sign, and

$$n = 2 \text{sgn}[\delta_o] \left[\sum_{i=1}^m (-1)^{i-1} \text{sgn}[q(\omega_{oi})] \right], n = 2m \forall m \in I_+$$

$$n = 2 \text{sgn}[\delta_o] \left[\sum_{i=1}^{m+1} (-1)^{i-1} \text{sgn}[q(\omega_{oi})] \right], n = 2m + 1 \forall m \in I_+, \omega_{om+1} = \infty$$

The classical Hermite-Biehler Theorem fails to provide any further information when the polynomial is not Hurwitz. However, the generalized Hermite-Biehler Theorem (Datta et al., 1999; Ho et al., 2000) ascertains Hurwitz stability and at the same time provides information about the number of RHP roots, if any.

Theorem 3 (Generalized Hermite-Biehler Theorem): Let

$\delta(s) = \sum_{i=0}^n \delta_i s^i, \delta_i \in \mathbb{R} \forall i$ with a root at the origin of multiplicity k . Let $0 < \omega_{o1} < \omega_{o2} \dots < \omega_{om-1}$ be the zeros of $q(\omega)$ that are real, distinct and nonnegative. Also, define $\omega_o = 0, \omega_{om} = \infty$, and $p^{(k)}(\omega_o) = \left(\frac{d^k}{d\omega^k} p(\omega) \right)_{\omega=\omega_o}$.

Then,

$$\sigma[\delta(s)] = (-1)^{m-1} \left[\text{sgn}[p^{(k)}(\omega_o)] + 2 \sum_{i=1}^{m-1} (-1)^i \text{sgn}[p(\omega_{oi})] + (-1)^m \text{sgn}[p(\omega_{om})] \right] \text{sgn}[q(\infty)], n = 2m$$

$$\sigma[\delta(s)] = (-1)^{m-1} \left[\text{sgn}[p^{(k)}(\omega_o)] + 2 \sum_{i=1}^{m-1} (-1)^i \text{sgn}[p(\omega_{oi})] \right] \text{sgn}[q(\infty)], n = 2m + 1 \forall m \in I_+$$

where $q(\omega)$ is defined in Theorem 2, and $\sigma[\delta(s)] = n_\delta^{(L)} - n_\delta^{(R)}$ $n_\delta^{(L)}$ and $n_\delta^{(R)}$ denote the number of open left-half plane (LHP) and RHP roots of $\delta(s)$. The proof of the theorems can be found in the literature (Datta et al., 1999; Ho et al., 2000).

A Framework for Controller Synthesis.--Based on Hermite-Biehler Theorems, the following procedure is adopted for PID controller design of the unity gain feedback

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system. The plant and controller transfer functions are given as $G_p(s) = n_p(s)/d_p(s)$, $G_c(s) = n_c(s)/d_c(s)$, where $n_c(s) = K_d s^2 + K_p s + K_i$, $d_c(s) = s$. Let

$$\delta(s) = \psi(s)n_p(-s) = sd_p(s)n_p(-s) + (K_i + K_d s^2)n_p(s)n_p(-s) + K_p s n_p(s)n_p(-s) \quad (4)$$

Then, it can be verified that $\sigma[\delta(s)] = \sigma[\psi(s)] - \sigma[n_p(s)]$. Let $m_\psi = \Theta[\psi(s)]$ denote the order of $\psi(s)$, then the number of RHP poles of $\psi(s)$ is given by $n_\psi^{(R)} = \frac{1}{2}\{m_\psi - \sigma[\psi(s)]\}$.

It is easy to see that, if $\psi(s)$ is to be Hurwitz, then $\sigma[\psi(s)] = m_\psi$, and $\sigma[\delta(s)] = m_\psi - \sigma[n_p(s)]$. Note that $\delta(j\omega) = p(\omega) + jq(\omega)$ where the polynomials are $p(\omega) = p_1(\omega) + (K_i - K_d \omega^2)p_2(\omega)$ and $q(\omega) = q_1(\omega) + K_p q_2(\omega)$. Therefore, let $m_q = \Theta[q(\omega)]$ and $\tilde{m}_q \leq m_q$ be the number of distinct, nonnegative, real zeros of $q(\omega)$ for some value of $K_p \in (-\infty, \infty)$, and define

$$\tilde{I}_i = \text{sgn}[p_1(\omega_{oi}) + (K_i - K_d \omega_{oi}^2)p_2(\omega_{oi})] \in \{-1, 1\} \forall i \in [1, \tilde{m}_q],$$

$$\tilde{I}_0 = \text{sgn}[p_1(0) + K_i p_2(0)] \in \{-1, 0, 1\},$$

$$\rho = m_\psi - \sigma[n_p(s)], \gamma = (-1)^{\tilde{m}_q - 1} \text{sgn}[q(\infty)], \text{ and}$$

$$\alpha = \frac{1}{2}\{m_\psi + \Theta[n_p(s)]\}.$$

Then, if $\psi(s)$ is to be Hurwitz, there must be a feasible solution to either of the following equations (Roy and Iqbal, 2002; Datta et al., 1999; Ho et al., 2000):

$$\rho = \left[\tilde{I}_0 + 2 \sum_{i=1}^{\tilde{m}_q - 1} (-1)^i \tilde{I}_i + (-1)^{\tilde{m}_q} \tilde{I}_{\tilde{m}_q} \right] \gamma, \alpha \in I_+ \text{ and} \quad (5)$$

$$\rho = \left[\tilde{I}_0 + 2 \sum_{i=1}^{\tilde{m}_q - 1} (-1)^i \tilde{I}_i \right] \gamma, \alpha \notin I_+. \quad (6)$$

Assuming that a solution to equations (5) or (6) can be found, we denote this feasible set as $g^* = \{\tilde{I}_i\} \neq \emptyset \forall i \in [0, \tilde{m}_q]$.

Then, a non-empty stabilizing PID set

$S_{\tilde{\kappa}} = \{K_p, [K_i, \bar{K}_i], [K_d, \bar{K}_d]\}$ for some $K_p \in (-\infty, \infty)$ can be computed from a set of linear inequalities developed as $(g^*) \cdot \{[P_1] + [P_2][\kappa]\} > 0$, i.e., $\text{sgn}\{[P_1] + [P_2][\kappa]\} = g^*$, where the matrices $[P_1] \in \mathfrak{R}^{m_\psi \times 1}$, $[P_2] \in \mathfrak{R}^{m_\psi \times 2}$ and $[\kappa] \in \mathfrak{R}^{2 \times 1}$ are defined as

$$[P_1] = [p_1(0) \quad p_1(\omega_{o1}) \quad p_1(\omega_{o2}) \quad \dots \quad p_1(\omega_{o\tilde{m}_q-1})]^T \quad (7)$$

$$[P_2] = \begin{bmatrix} p_2(0) & p_2(\omega_{o1}) & p_2(\omega_{o2}) & \dots & p_2(\omega_{o\tilde{m}_q-1}) \\ 0 & -\omega_{o1}^2 p_2(\omega_{o1}) & -\omega_{o2}^2 p_2(\omega_{o2}) & \dots & -\omega_{o\tilde{m}_q-1}^2 p_2(\omega_{o\tilde{m}_q-1}) \end{bmatrix}^T, \text{ and} \quad (8)$$

$$[\kappa] = [K_i \quad K_d]^T, \quad S_{\tilde{\kappa}} \in \mathfrak{R} \quad (9)$$

Note that in the preceding analysis $S_{\tilde{\kappa}} \in \mathfrak{R}^+$, has been assumed. In practice, to facilitate tuning, we may require that $S_{\tilde{\kappa}} \in \mathfrak{R}^+ \mid \{(g^*) \cdot \{[P_1] + [P_2][\kappa]\} > 0\} \neq \emptyset$.

Necessary Conditions for Stability.--In this section we shall determine the necessary condition for the existence of K. To this effect, re-write equation (1) as

$$G_p(s) = \frac{n_p(s)}{d_p(s)} = \frac{a_1 s + a_0}{s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}. \quad (10)$$

Define $\{a\} = \{a_0, a_1\}$ and $\{b\} = \{b_0, b_1, b_2, b_3\}$. Note that for the given problem $m_\psi = 5$, $\rho = 4$, $\alpha = 3$, and the polynomial

$$q(\omega) = \omega \tilde{q}(\omega) = \frac{a_1}{a_0} [a_0 - b_3 a_1] \omega^5 + \{(b_1 a_1 - b_2 a_0) + K_p a_1^2\} \omega^3 + (b_0 a_0 + a_0^2) \omega \quad (11)$$

Using equation (11) we determine that $q(\infty) = \frac{a_1(a_0 - b_3 a_1)}{a_0}$ such that $(\gamma = (-1)^{\tilde{m}_q - 1} s_0$ where $s_0 = \text{sgn} \left[\frac{a_1(a_0 - b_3 a_1)}{a_0} \right]$; then,

if $\psi(s)$ is to be Hurwitz stable, we must have (Roy and Iqbal, 2002; Ho et al., 2000):

$$(-1)^{\tilde{m}_q - 1} \left[\tilde{I}_0 + 2 \sum_{i=1}^{\tilde{m}_q - 1} (-1)^i \tilde{I}_i + (-1)^{\tilde{m}_q} \tilde{I}_{\tilde{m}_q} \right] s_0 = \rho \quad (12)$$

where $\tilde{I}_i = \begin{cases} \tilde{I}_0 \in \{-1, 0, 1\}, i = 0 \\ \tilde{I}_i \in \{-1, 1\}, i \neq 0 \end{cases}$ and \tilde{m}_q is the number of roots

$q(\omega, K_p)$ that are real, nonnegative, and distinct, given as $\omega_0 = 0$, and the four roots of the equation $\tilde{q}(\omega, K_p) = 0$ are given as

$$\omega_{1,2,3,4}^2 = \lambda_{1,2} = \frac{b_2 a_0 - b_1 a_1 - K_p a_1^2 \pm \sqrt{\Delta}}{2(a_0 - b_3 a_1)} \quad (13)$$

where $\lambda = \omega^2$ and define

$$\Delta(K_p) = \{(b_1 a_1 - b_2 a_0) + K_p a_1^2\}^2 - 4a_0(b_0 + a_0)(a_0 - b_3 a_1).$$

Let $0 \leq \tilde{m}^* \leq 4$ be the number of roots of $\tilde{q}(\omega, K_p) = 0$ that are real, nonnegative, and distinct; then the stability results for the various cases of \tilde{m}_q are summarized in Table 1. The necessary conditions for stability of the closed-loop system defined by equations (1) and (2) are given by the following theorem.

Theorem 4: Let Ω be a logical set defined as $\Omega = \text{sgn}[\Delta] \neq -1 \wedge \sum_i \text{sgn}[\lambda_i] \geq 0$; then, the necessary condition for a set of stabilizing PID controllers for the process given by (1) is $\Omega \neq \emptyset$.

Proof: From Table 1, the conditions for $g^* \neq \emptyset$ are

$$\tilde{m}^* = \begin{cases} 1 \Leftrightarrow \text{sgn}[\Delta] \neq -1 \wedge \prod_i \text{sgn}[\lambda_i] = -1 \\ 2 \Leftrightarrow \text{sgn}[\Delta] \neq -1 \wedge \prod_i \text{sgn}[\lambda_i] \geq 0. \end{cases}$$

Combining the two conditions and recognizing that

$$\prod_i \text{sgn}[\lambda_i] = -1 \vee \prod_i \text{sgn}[\lambda_i] \geq 0 \equiv \sum_i \text{sgn}[\lambda_i] \geq 0,$$

we obtain the result $g^* \neq \emptyset \Leftrightarrow \Omega \neq \emptyset$.

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Q.E.D.

The following lemmas illustrate how the necessary conditions are determined for the problem.

Lemma 1: $\text{sgn}[\Delta] \neq -1$ if and only if $K_p \in S^*(K_p)$ where Δ is as defined above, and

$$S^*(K_p) = \begin{cases} (-\infty, \min\{k_{1,2}^*\}) \cup [\max\{k_{1,2}^*\}, \infty), & k_{1,2}^* \in \mathfrak{R} \\ (-\infty, \infty), & k_{1,2}^* \in C \end{cases}$$

where $k_{1,2}^*$ are the roots of Δ .

Proof: From equation (13), we note that we can write Δ as a quadratic in K_p ,

$$\Delta = a_1^4 K_p^2 + 2a_1^2(b_1 a_1 - b_2 a_0) K_p + c = \prod_i (K_p - k_i^*), \text{ where}$$

$c = (b_1 a_1 - b_2 a_0)^2 - 4a_0(b_0 + a_0)(a_0 - b_3 a_1)$. Then, it can be seen that $\Delta \geq 0 \forall K_p \in (-\infty, \min\{k_{1,2}^*\}) \cup [\max\{k_{1,2}^*\}, \infty)$ and

$\Delta < 0 \forall K_p \in (\min\{k_{1,2}^*\}, \max\{k_{1,2}^*\})$. Therefore, $\text{sgn}[\Delta] \neq -1 \forall K_p \in (-\infty, \min\{k_{1,2}^*\}) \cup [\max\{k_{1,2}^*\}, \infty) = S^*(K_p)$.

However, it can be verified that for $\forall k_{1,2}^* \in C$,

$$\Delta > 0 \forall K_p \Rightarrow \text{sgn}[\Delta] \neq -1 \Rightarrow S^*(K_p) = (-\infty, \infty).$$

Q.E.D.

Lemma 2: For $\sum_i \text{sgn}[\lambda_i] \geq 0$ the range of K_p is given by

the following range $\{K_p \in \bar{S}(K_p)\} \vee K_p \in \begin{cases} \tilde{S}(K_p) & \text{for } s_1 = -1 \\ \emptyset & \text{for } s_1 = 1 \end{cases}$

$$\text{where } \bar{S}(K_p) = \left(-\infty, \frac{b_1|a_1| - b_2|a_0|}{a_1^2}\right), \tilde{S}(K_p) = \left(\frac{b_1|a_1| - b_2|a_0|}{a_1^2}, \infty\right),$$

$$\text{and } s_1 = \text{sgn} \left[\frac{a_0(b_0 + a_0)}{(a_0 - b_3 a_1)} \right].$$

Proof: Note that

$$\lambda_{1,2} = \frac{b_2 a_0 - b_1 a_1 - K_p a_1^2 \pm \sqrt{\Delta}}{2(a_0 - b_3 a_1)} = \frac{B(K_p) \pm \sqrt{\Delta}}{A(\{a\}, \{b\})} = \bar{B}_1(K_p) \pm \bar{B}_2(K_p) \quad (14)$$

$$\text{where } \bar{B}_1(K_p) = \frac{B_1(K_p)}{A}, \bar{B}_2(K_p) = \frac{\sqrt{\Delta}}{A}.$$

Then, regardless of the value of $\text{sgn}[\bar{A}]$, since

$$\Delta \geq 0 \forall K_p \in S^*(K_p),$$

$$\sum_i \text{sgn}[\lambda_i] \geq 0 \equiv \text{sgn}[\bar{B}_1] = 1 \vee \{ \text{sgn}[\bar{B}_1] = -1 \wedge \text{sgn}[|\bar{B}_1| - |\bar{B}_2|] = -1 \}$$

But $\text{sgn}[\bar{B}_1] = \text{lif}$ and only if $K_p \in \bar{S}(K_p)$, while

$\text{sgn}[\bar{B}_1] = -1 \Leftrightarrow K_p \in \tilde{S}(K_p)$. However,

$$\text{sgn}[|\bar{B}_1| - |\bar{B}_2|] = -1 \equiv \text{sgn}[\bar{B}_1^2 - \bar{B}_2^2] = \text{sgn} \left[\frac{a_0(b_0 + a_0)}{(a_0 - b_3 a_1)} \right] = s_1 = -1.$$

Since s_1 is a function that is independent of K_p , we have

$$\{K_p \mid \text{sgn}[|\bar{B}_1| - |\bar{B}_2|] = -1\} = \begin{cases} (-\infty, \infty) & \text{for } s_1 = -1 \\ \emptyset & \text{for } s_1 = 1 \end{cases} \text{ or,}$$

$$\text{sgn}[\bar{B}_1] = -1 \wedge \text{sgn}[|\bar{B}_1| - |\bar{B}_2|] = -1 \Rightarrow \{K_p \in \bar{S}(K_p)\} \vee K_p \in \begin{cases} S(K_p) & \text{for } s_1 = -1 \\ \emptyset & \text{for } s_1 = 1 \end{cases}$$

Q.E.D.

Theorem 5 (Necessary Condition): The process given by equation (1) is stabilizable, i.e., $\exists g^* \neq \emptyset$ if and only if $K_p \in S(K_p)$ where is evaluated as

$$(i) S(K_p) = \begin{cases} S^*(K_p) & \text{for } s_1 = -1 \\ S^*(K_p) \cap \bar{S}(K_p) & \text{for } s_1 = 1 \end{cases} \quad \forall k_{1,2}^* \in \mathfrak{R},$$

$$(ii) S(K_p) = \begin{cases} (-\infty, \infty) & \text{for } s_1 = -1 \\ \bar{S}(K_p) & \text{for } s_1 = 1 \end{cases} \quad \forall k_{1,2}^* \in C$$

where $S^*(K_p)$ and $\bar{S}(K_p)$ are as defined in Lemma 1 and Lemma 2.

Proof: (i) From Lemma 1 and Lemma 2 we get

$$\text{sgn}[\Delta] \neq -1 \wedge \sum_i \text{sgn}[\lambda_i] \geq 0 \equiv K_p \in S^*(K_p) \cap \bar{S}(K_p) \cup \begin{cases} \tilde{S}(K_p) & \text{for } s_1 = -1 \\ \emptyset & \text{for } s_1 = 1 \end{cases}$$

Substituting the above condition in the result of Theorem 4, we obtain

$$g^* \neq \emptyset \Leftrightarrow K_p \in S^*(K_p) \cap \bar{S}(K_p) \cup \begin{cases} \tilde{S}(K_p) & \text{for } s_1 = -1 \\ \emptyset & \text{for } s_1 = 1 \end{cases}$$

$$\Rightarrow S(K_p) = \begin{cases} S^*(K_p) \cap \bar{S}(K_p) \cup \tilde{S}(K_p) & \text{for } s_1 = -1 \\ S^*(K_p) \cap \bar{S}(K_p) & \text{for } s_1 = 1 \end{cases}$$

Since $\bar{S}(K_p) \cup \tilde{S}(K_p) = (-\infty, \infty) = \mathfrak{R}$ and

$S^*(K_p) \cap (-\infty, \infty) = S^*(K_p)$, we obtain

$$S(K_p) = \begin{cases} S^*(K_p) & \text{for } s_1 = -1 \\ S^*(K_p) \cap \bar{S}(K_p) & \text{for } s_1 = 1 \end{cases}$$

(ii) Observe from Lemma 1 that $\forall k_{1,2}^* \in C, S^*(K_p) = (-\infty, \infty)$.

Using this result in (i), and recognizing that

$(-\infty, \infty) \cap \bar{S}(K_p) = \bar{S}(K_p)$, the result follows. Q.E.D.

Sufficient Conditions for Stability.—We now determine the sufficient conditions for the existence of the stabilizing PID set. We note that sufficient conditions for the PID to exist are given by (Roy and Iqbal, 2002; Ho et al., 2000)

$$(g^*) \cdot \{[P_1] + [P_2][\tilde{\kappa}]\} > 0 \neq \emptyset \quad (15)$$

where the real matrices $[P_1]$, $[P_2]$, and $[\tilde{\kappa}]$ are defined by

equations (7)-(9). From Table I we know that

$g^* \neq \emptyset \Leftrightarrow \tilde{m}_q = 2, 3$. Define

$$f = \frac{a_1 \max(\lambda_i)^3 + (a_0 b_3 - a_1 b_2) \max(\lambda_i)^2 + (|K_i| a_1^2 - a_0 b_1) \max(\lambda_i) + |K_i| a_0^2}{\max(\lambda_i)(a_1^2 \max(\lambda_i) + a_0^2)}$$

$$\bar{f} = \frac{a_1 \max(\lambda_i)^3 + (a_0 b_3 - a_1 b_2) \max(\lambda_i)^2 - (|K_i| a_1^2 - a_0 b_1) \max(\lambda_i) - |K_i| a_0^2}{\max(\lambda_i)(a_1^2 \max(\lambda_i) + a_0^2)}$$

$$\eta_{1,2} = \frac{a_1 \lambda_{1,2}^3 + (a_0 b_3 - a_1 b_2) \lambda_{1,2}^2 + (|K_i| a_1^2 - a_0 b_1) \lambda_{1,2} + |K_i| a_0^2}{\lambda_{1,2}(a_1^2 \lambda_{1,2} + a_0^2)}$$

and $S_\eta = (\min\{\eta_{1,2}\}, \max\{\eta_{1,2}\})$, $S_\pi = (\min\{\pi_{1,2}\}, \max\{\pi_{1,2}\})$

The following lemma develops the sufficiency condition.

Lemma 3: A stabilizing PID set $\tilde{\kappa}$, for the process in equation (10) exists only under the following conditions: for a given $K_p \in S(K_p)$,

- 1) If $\tilde{m}^* = 1$, then $\begin{cases} K_i \in (0, \infty), K_d \in (f, \infty) \text{ when } \gamma = 1 \\ K_i \in (-\infty, 0), K_d \in (-\infty, \bar{f}) \text{ when } \gamma = -1. \end{cases}$
- 2) If $\tilde{m}^* = 2$, then
- (a) If $\sum_i \text{sgn}[\lambda_i] = 1$, then $\begin{cases} K_i \in (0, \infty), K_d \in (f, \infty) \text{ when } \gamma = 1 \\ K_i \in (-\infty, 0), K_d \in (-\infty, \bar{f}) \text{ when } \gamma = -1. \end{cases}$
- (b) If $\sum_i \text{sgn}[\lambda_i] = 2$, then $\begin{cases} K_i \in \mathfrak{R}^+, K_d \in S_\eta \text{ when } \gamma = 1 \\ K_i \in \mathfrak{R}^-, K_d \in S_\pi \text{ when } \gamma = -1. \end{cases}$

Proof: 1) From Table 1, for $\tilde{m}^* = 1$, $\text{sgn}[\Delta] \neq -1 \wedge \prod_i \text{sgn}[\lambda_i] = -1 \equiv \text{sgn}[\Delta] \neq -1 \wedge \sum_i \text{sgn}[\lambda_i] = 0$.

But from Theorem 4 and Theorem 5, we have

$$\text{sgn}[\Delta] \neq -1 \wedge \sum_i \text{sgn}[\lambda_i] \geq 0 \forall K_p \in S(K_p).$$

Therefore, whenever $\tilde{m}^* = 1$, $\sum_i \text{sgn}[\lambda_i] \geq 0 \forall K_p \in S(K_p)$.

Using this result in equation (13), we deduce $\omega_{o1} = \sqrt{\max(\lambda_i)}$.

Furthermore, from equations (5)-(7) and the set of linear inequalities, we obtain

$$\begin{cases} K_i > 0, K_d > \frac{p_1(\omega_{o1}) + |K_i| p_2(\omega_{o1})}{\max(\lambda_i) p_2(\omega_{o1})} \text{ when } \gamma = 1 \\ K_i < 0, K_d < \frac{p_1(\omega_{o1}) - |K_i| p_2(\omega_{o1})}{\max(\lambda_i) p_2(\omega_{o1})} \text{ when } \gamma = -1. \end{cases}$$

Substituting ω_{o1} in the expressions of $p_{1,2}(\omega)$ (Datta et al., 1999), we obtain

$$\frac{p_1(\omega_{o1}) + |K_i| p_2(\omega_{o1})}{\max(\lambda_i) p_2(\omega_{o1})} = f \text{ and } \frac{p_1(\omega_{o1}) - |K_i| p_2(\omega_{o1})}{\max(\lambda_i) p_2(\omega_{o1})} = \bar{f}.$$

2a.) If $\sum_i \text{sgn}[\lambda_i] = 1$, then $\{\omega_i\} = \{0, 0, 2\Re \neq 0\}$. Then the root distribution of $q(\omega, K_p)$ is $\{0, 0, 0, 2\Re \neq 0\}$.

Therefore the roots that qualify for the computation of equation (15) are $\{\omega_0 = 0, \omega_{o1} = \sqrt{\max(\lambda_i)}\}$, and the proof follows from equation (1) of Lemma 3.

2b.) The proof is on similar lines as (2a) except that for $\sum_i \text{sgn}[\lambda_i] = 2$, the dimensions in equations (5) and (6) increase by one. It is known from equation (13) that for $\sum_i \text{sgn}[\lambda_i] = 2$, $\omega_{o1,2} = \sqrt{\lambda_{1,2}}$. The result follows from (2a) in conjunction with equation (15) and making substitution for $\omega_{o1,2}$.

Q.E.D.

The stability results of Lemmas 1, 2, and 3 are summarized in the following theorem.

Theorem 6 (Main result on stabilization): The process given by equation (10) is stabilizable with a PID controller if and only if $K_p \in S(K_p)$ and if the ranges of K_i, K_d are chosen according to the following situations:

- (a) If $\sum_i \text{sgn}[\lambda_i] = 0, 1$ and $\gamma = 1$, then $K_i \in \mathfrak{R}^+, K_d \in (f, \infty)$.
- (b) If $\sum_i \text{sgn}[\lambda_i] = 0, 1$ and $\gamma = -1$, then $K_i \in \mathfrak{R}^-, K_d \in (-\infty, \bar{f})$.
- (c) If $\sum_i \text{sgn}[\lambda_i] = 2$ and $\gamma = 1$, then either $(K_i \in \mathfrak{R}^+, K_d \in S_\eta)$, or $(K_i \in \mathfrak{R}^-, K_d \in S_\pi)$ whichever is non-empty.
- (d) If $\sum_i \text{sgn}[\lambda_i] = 2$ and $\gamma = -1$, then $(K_i \in \mathfrak{R}^+, K_d \in S_\eta)$, or $(K_i \in \mathfrak{R}^-, K_d \in S_\pi)$ whichever is non-empty.

Proof: For closed-loop stability we know that the necessary condition for the existence of a stabilizing PID controller is given by Theorem 5, i.e., $K_p \in S(K_p)$. Furthermore, (a) and (b) are the results of Lemma 3(1) and 3(2a), while (c) and (d) follow from Lemma 3(2b). Q.E.D.

The following algorithm is proposed to determine the stabilizing PID set for the problem:

- (i) Use the Theorem 5 to calculate $S(K_p)$. Constraint the value of K_p to $K_p = K_{p0} \in S(K_p)$.
- (ii) Calculate the value of $\sum_i \text{sgn}[\lambda_i]$ and γ . Note that γ is given as

$$\gamma = (-1)^{m_q - 1} s_o = \begin{cases} -s_o & \text{for } \sum_i \text{sgn}[\lambda_i] = 0, 1 \\ s_o & \text{for } \sum_i \text{sgn}[\lambda_i] = 2 \end{cases}$$

Accordingly, calculate f (or \bar{f}) or S_η (or S_π).

- (iii) Use Theorem 6 to evaluate the stabilizing PID set.

Remark 1: From Theorem 6 and the above algorithm, we can write the stabilizing PID set as

$$\forall K_p = K_{p0} \in S(K_p),$$

(a) $S_\varepsilon = \{K_{p0}, \mathfrak{R}^+, (f, \infty)\}$ or $S_\varepsilon = \{K_{p0}, \mathfrak{R}^-, (-\infty, \bar{f})\}$ for $\sum_i \text{sgn}[\lambda_i] = 0, 1$ and $\gamma = \pm 1$.

(b) $S_\varepsilon = \{K_{p0}, \mathfrak{R}^+, S_\eta\}$ or $S_\varepsilon = \{K_{p0}, \mathfrak{R}^-, S_\pi\}$ for $\sum_i \text{sgn}[\lambda_i] = 2$ and $\gamma = \pm 1$.

Remark 2: Note that f, \bar{f}, η , and π are a linear function of K_i as,

$$f, \bar{f} = \pm \frac{(a_1^2 \max(\lambda_i) + a_0^2)}{\max(\lambda_i)(a_1^2 \max(\lambda_i) + a_0^2)} |K_i| + \frac{a_1 \max(\lambda_i)^3 + (a_0 b_3 - a_1 b_2) \max(\lambda_i)^2 - a_0 b_1 \max(\lambda_i)}{\max(\lambda_i)(a_1^2 \max(\lambda_i) + a_0^2)}$$

$$\eta_{1,2}, \bar{\eta}_{1,2} = \pm \frac{(a_1^2 \lambda_{1,2} + a_0^2)}{\lambda_{1,2}(a_1^2 \lambda_{1,2} + a_0^2)} |K_i| + \frac{a_1 \lambda_{1,2}^3 + (a_0 b_3 - a_1 b_2) \lambda_{1,2}^2 - a_0 b_1 \lambda_{1,2}}{\lambda_{1,2}(a_1^2 \lambda_{1,2} + a_0^2)}$$

Therefore, a constrained point value of $K_i \in \mathfrak{R}^\pm$ generates an interval of K_d , and the bounds of the interval depend linearly on K_i . This concept is illustrated in Figure 3.

Results

Example 1. We consider the arm-manipulator problem when the process parameters are given as $\tilde{A} = \{a, b, k, c, m, \xi, \omega_n\} = \{-10^3, -10^2, 101, 10, 1, 0.25, 2\}$

and the process is controlled by a PID controller. Therefore, $\{a\} = \{-400, -4000\}$ and $\{b\} = \{404, 141, 115, 11\}$. Then $s_1 = -1$ and $\Delta = 10^{11} (2560K_p^2 - 165.76K_p + 2.68) \Rightarrow k_{1,2}^* \in C$. Using Lemma 1, $S^*(K_p) = (-\infty, \infty)$, and from Theorem 5, $S(K_p) = S^*(K_p) = (-\infty, \infty)$. Let $K_p = K_{p0} = 1$, then $\Delta = 2.3969 \times 10^{14}$. From equation (13) we obtain $\lambda_1 = -355.09$, $\lambda_2 = 1.04 \times 10^{-4} \Rightarrow \sum_i \text{sgn}[\lambda_i] = 0$, and \max

$(\lambda_i) = 1.04 \times 10^{-4}$. Therefore, $\tilde{m}_q = 2 \Rightarrow \gamma = (-1)^1 \text{sgn}\{436000\} = -1$. Then from Theorem 6, $K_i \in \mathfrak{R}^- = (-\infty, 0)$ and $K_d \in (-\infty, \tilde{f})$ where the value of \tilde{f} depends on the constrained value of K_i . The stabilizing PID set is given by $S_\kappa = \{1, (-\infty, 0), (-\infty, \tilde{f})\}$. For example, constraining the value of $K_i = -1$, we obtain $\tilde{f} = -9615.03$ and from Lemma 3, the stabilizing PID set is given by $S_\kappa = \{1, -1, (-\infty, -9615.03)\}$. As a further example, the closed-loop system is simulated for $\tilde{\kappa} = \{1, -1, -10^4\}$, and the response is shown in Figure 4.

Example 2. ($\tilde{A} > 0$) Consider a manipulator-sensor process given by equation (1) with a process parameter set $\tilde{A} = \{10^3, 10^2, 101, 10, 1, 0.25, 2\}$. Then $s_1 = s_0 = -1$, $\Delta = 10^{11} (2560K_p^2 + 165.76K_p + 3.2441) \Rightarrow k_{1,2}^* \in C$. From Lemma 2, $S^*(K_p) = (-\infty, \infty)$, and from Theorem 5, we find that $S(K_p) = S^*(K_p) = (-\infty, \infty)$. Let $K_p = K_{p0} = 1$, then $\Delta = 2.7290 \times 10^{14}$. From equation (13) we obtain $\lambda_1 = -378.87$, $\lambda_2 = 0.0229 \Rightarrow \sum_i \text{sgn}[\lambda_i] = 0$, and \max

$(\lambda_i) = \lambda_2 = 0.0229$. Thus $\tilde{m}_q = 2 \Rightarrow \gamma = (-1)^1 s_0 = 1$. From Theorem 6 we obtain $K_i \in \mathfrak{R}^+ = (0, \infty)$ and $K_d \in (\tilde{f}, \infty)$ where the value of \tilde{f} similarly depends on the constrained value of K_i . The stabilizing PID set is given by $S_\kappa = \{1, (0, \infty), (\tilde{f}, \infty)\}$. Constraining the value of $K_i = 1$, we obtain $\tilde{f} = 3.1199$ and from Lemma 3, the stabilizing PID set is given by $S_\kappa = \{1, 1, (3.1199, \infty)\}$. The closed-loop system is simulated for $\tilde{\kappa} = \{1, 1, 4\}$ and is shown in Figure 5.

Example 3. ($k, c < 0$) Let an unstable manipulator-sensor process be given by the following parameters: $A = \{10^3, 10^2, -101, -10, 1, 0.25, 2\}$. Then $s_1 = 1$, $s_0 = -1$, and $\Delta = 10^{11} (2560K_p^2 - 166.78K_p + 2.7137) \Rightarrow k_{1,2}^* \in \mathfrak{R}$. From Lemma 1 and Lemma 2, $S^*(K_p) = (-\infty, 0.0336) \cup (0.0316, \infty)$ and $\bar{S}(K_p) = (-\infty, -0.0326)$. Furthermore, from Theorem 5 we obtain $S(K_p) = S^*(K_p) \cap \bar{S}(K_p) = (-\infty, -0.0326)$. Let $K_p = K_{p0} = -1$, then $\Delta = 2.7290 \times 10^{14}$. From equation (13) we obtain $\lambda_1 = -2.17$, $\lambda_2 = 381.10 \Rightarrow \sum_i \text{sgn}[\lambda_i] = 0$, and

$\max(\lambda_i) = \lambda_2 = 381.10$. Therefore, $\tilde{m}_q = 2 \Rightarrow \gamma = (-1)^1 s_0 = 1$. From Theorem 6, $K_i \in \mathfrak{R}^+ = (0, \infty)$ and $K_d \in (f, \infty)$ where the value of f depends on the constrained value of K_i . The stabilizing PID set is given by $S_\kappa = \{-1, (0, \infty), (f, \infty)\}$. Constraining the value of $K_i = 1$, we obtain $f = 0.1249$ and, from Lemma 3, the stabilizing PID set given by $\{-1, 1, (0.1249, \infty)\}$. The closed-loop system is simulated for $\tilde{\kappa} = \{-1, 1, 0.5\}$ and is shown in Figure 6.

Conclusions

In conclusion, this paper discusses PID controller stabilization of a robot manipulator equipped with a wrist sensor in an unstructured work environment. The problem is formulated in a general framework that can also be used for other similar applications. Necessary and sufficient conditions for stability are derived using the analytical framework of Hermite-Biehler Theorem that is based on the interlacing property of the even and odd parts of the characteristic polynomial. The controller synthesis involves solution of a set of linear matrix inequalities (LMIs) that can be solved on the computer. We propose an algorithm that conveniently solves for controller parameters given the plant model. Our simulation results for the robot-manipulator examples show that the controller, when used in conjunction with the position-controlled manipulator, effectively reduces the arm effort during manipulation tasks. The approach, albeit conservative in terms of being model specific, is successful in identifying a set of all stabilizing PID controllers for the given problem.

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Table 1. Stability results for various cases of roots of $\tilde{q}(\omega, K_p)$

\tilde{m}^*	Possible $\{\omega_i\}$ for $g^* \neq \emptyset$	Condition on sign of Δ
0	$\{\emptyset\}$	Not applicable
1	$\{2\Re, 2C\}$	$\text{sgn}[\Delta] \neq -1 \wedge \prod_i \text{sgn}[\lambda_i] = -1$
2	$\{4\Re \neq 0\}, \{0, 0, 2\Re \neq 0\}$	$\text{sgn}[\Delta] \neq -1 \wedge \prod_i \text{sgn}[\lambda_i] \geq 0$
3, 4	$\{\emptyset\}$	Not applicable

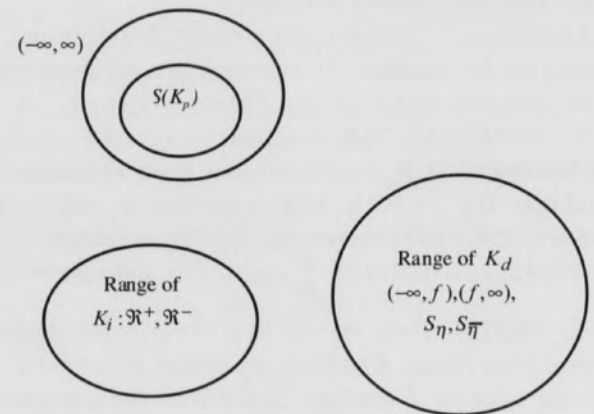


Fig. 3. A set diagram illustrating the concept of the algorithm derived from Theorem 6.

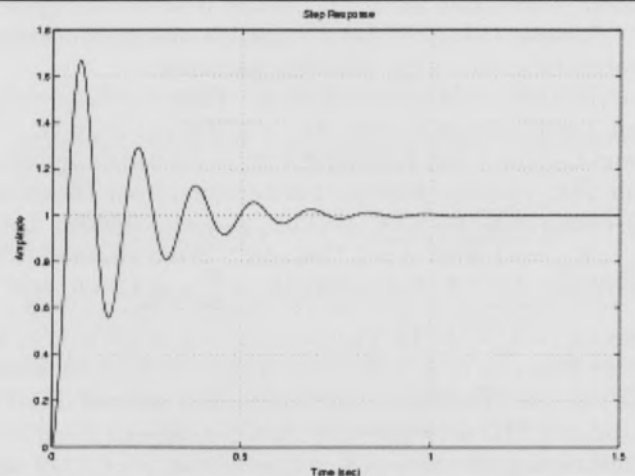


Fig. 4. Closed-loop step response with $\tilde{\kappa} = \{1, -1, -10^4\}$ when $a, b < 0$. The choice of parameters represents robot acting in collaboration with human arm.

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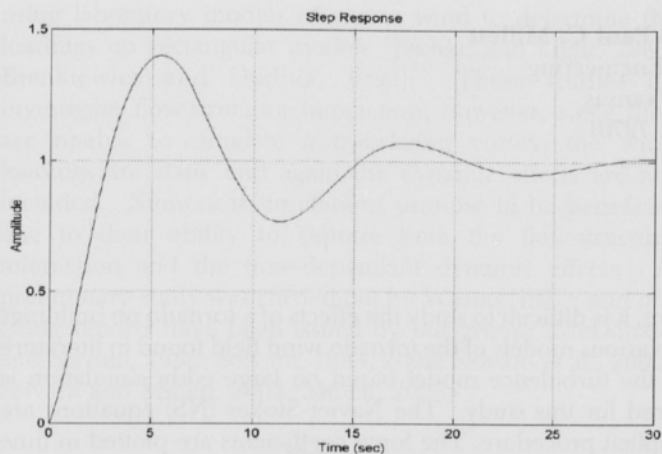


Fig. 5. Closed-loop step response with $\tilde{k} = \{1, 1, 4\}$ when $\tilde{A} > 0$. The choice of parameters represents robot manipulator alone.

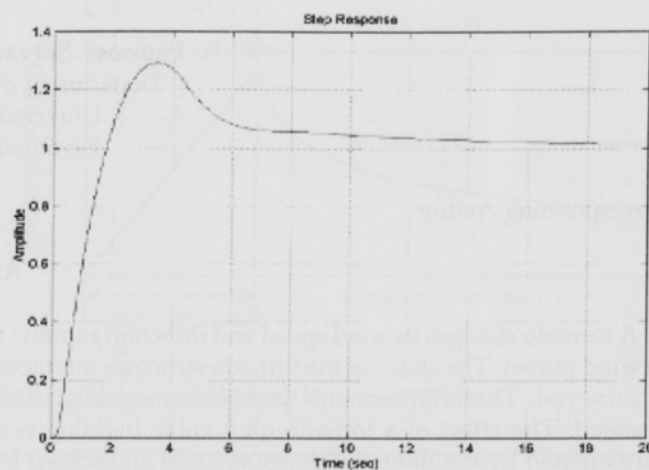


Fig. 6. Closed-loop step response with $\tilde{k} = \{-1, 1, 0.5\}$ when $k, c < 0$. The choice of parameters represents a possibly unstable sensor response.