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Abstract

Geographic Information Systems (GIS) offer a cost-effective way to analyze and inventory land and environmental resources. There are many attributes that can be displayed and analyzed in GIS. One of these attributes is slope, which can be calculated from a digital elevation model (DEM). Slope is an important factor in a variety of models used in land analysis as well as land use and management. There are several different mathematical computational algorithms used to calculate slope within a GIS. Eight different slope calculation methods were investigated in this study. These methods were used to calculate slope using 10-m, 30-m, and 100-m DEMs. There were two phases of analysis in this study. The first phase was a cell-by-cell comparison of the eight slope algorithms for all three DEMs to obtain an understanding of differences between the calculated slope methods. The second phase was to determine the method that calculated the most accurate slope from a 10-m, 30-m, and 100-m DEM, by comparing calculated slope to actual slope value. All methods underestimated slope for the 100-m DEM with a mean slope difference ranging from 9.28% to 11.085%. For the 30-meter DEMs all the slope methods underestimated slope, with a mean slope difference range from 0.21% to 4.18%. The 10-meter DEM mean slope difference ranged from –2.63% to 1.82% for the cell slope methods. For all methods, steeper slopes, greater than approximately 40%, were underestimated when slope was calculated from a DEM.

Introduction

Geographic Information Systems (GIS) offer a cost-effective way to analyze and inventory land and environmental resources (Goodchild and Palladino, 1995). As a result, GIS has become very popular with resource managers. Resource managers are able to inventory resources such as timber and wildlife habitat (Goodchild and Palladino, 1995). There are many attributes that can be analyzed and displayed in GIS. One such attribute is slope. Slope is the rate of change in altitude at a point on a surface and is often called gradient (Burrough, 1992). Slope is an important and widely used topographic attribute and can be calculated directly from a Digital Elevation Model (DEM).

Digital Elevation Models (DEMs) have been developed and provided to users for performing a wide variety of terrain analyses (Lee et al., 1992). DEMs come in different scales and spatial resolutions (grid spacings). Scale refers to the relationship between distance on a map and distance on the earth's surface. Spatial resolution is the area on the earth's surface represented by a cell of a grid. The 1:24,000 and 1:100,000 scales were used in this study. The resolutions used were10 m, 30 m, and 100 m. The 10-m and 30-m DEMs were 1:24,000 scale and the 100-m DEM was 1:100,000 scale. Due to the detail and availability of these DEMs, they are the most often used in GIS. For this reason, they were chosen for investigation in this study.

There are several different mathematical computational algorithms used to calculate slope from a DEM. Weih (1991) found that the results of these various methods differ, some varying by as much as 40%. Since slope is often a key attribute in environmental modeling, this variation poses a problem. If the slope method used for a particular model does not accurately reflect reality, then conclusions derived from that model may be incorrect. For example, Weih and Smith (1997) used a model to determine land suitable for timber production in Virginia. Within their model the only variable that changed was the slope method used. They found up to a 4.5 times difference in the amount of land deemed unsuitable for timber production.

There are four functional units in a typical GIS: data input, data model, data manipulation, and data presentation (Shekhar et al., 1997). As GIS have progressed from being a descriptive tool to a decision making tool, the errors and variability of the components in a GIS have become important (Weih and Smith, 1997). When manipulating data, it is important for the user to know which method(s) the GIS are using to perform a given task. Since many GIS lack information about how they should be used, the user often has little information on how to achieve the optimum

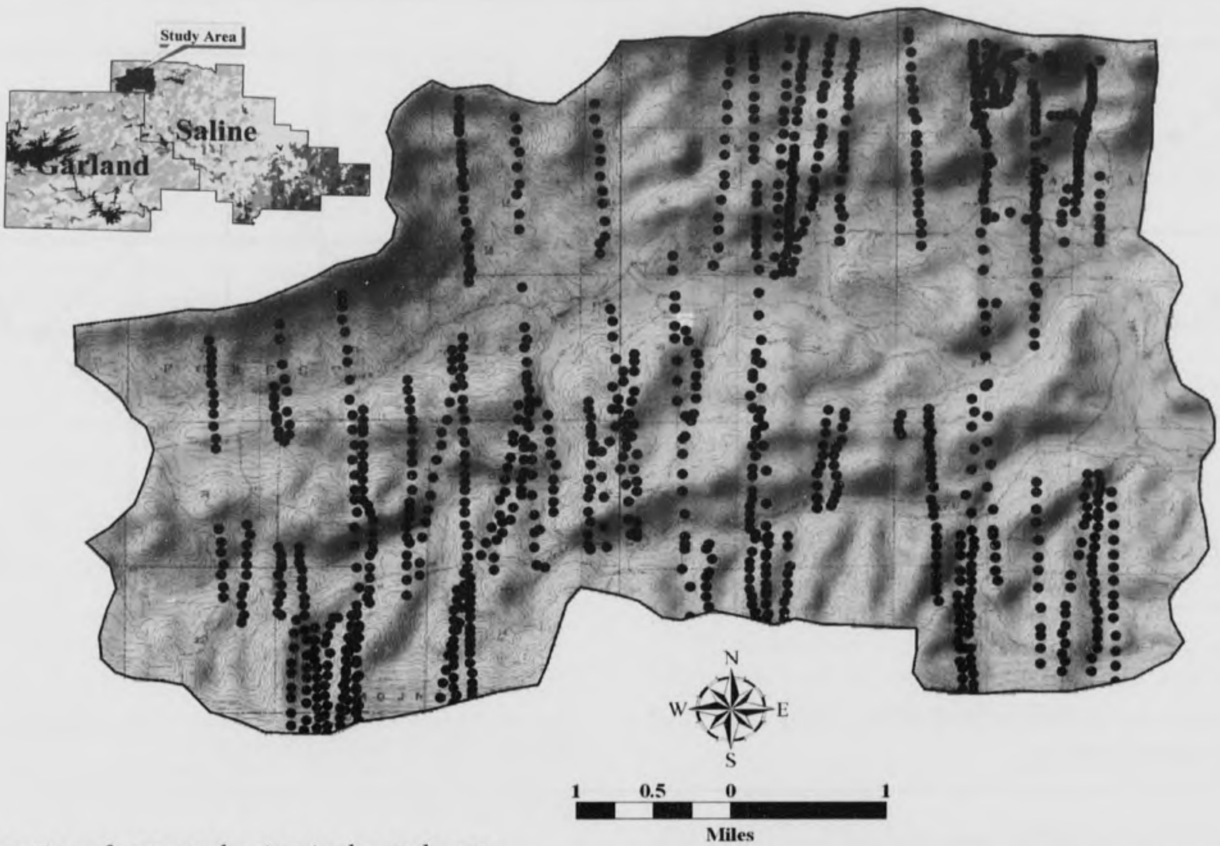

Fig. 1. Location of measured points in the study area.

results (Burrough, 1992). Unfortunately, the method used to determine slope is rarely specified in detail by the GIS software vendor. The practitioner may therefore be using a method to calculate slope from a DEM that is not optimal given his/her objectives. Also, GIS practitioners are not always aware of the effects of different methods on their results. Therefore, users should become aware of the types of errors that might exist in any spatial database.

Methods

The study area, approximately 70 square kilometers in size, was located in the Ouachita National Forest in Garland and Saline counties in central Arkansas. Fig. 1 shows the locations of the 1,200 points measured in this study. At each point, slope and latitude/longitude coordinates were recorded. Point slope measurements were taken approximately every 100 m along a north-south transect. Data collectors paced themselves to approximate this distance. The slope measurements (percent) were made using a Suunto clinometer. On flat terrain, the eye level of the data collector was marked on a pole. At each data collection point, the data collector placed the pole 10 feet away, based on the maximum slope, and took the upslope and downslope measurements by aiming the clinometer at the predetermined eye level. The average of these two slope measurements was recorded as the slope for that point.

The latitude/longitude coordinates were collected with 8 channel handheld GPS receivers. The receivers collected 120 positions, meaning they collected one position (latitude, longitude, and elevation) per second for 120 seconds. The recorded position was the average of these 120 positions, thereby providing a more accurate position. Using PC-GPS© (2.5 and 3.6) software, the GPS data were differentially corrected. There were 1130 points that could be used in this study after differential correction. Some data were lost due to the inability to differentially correct them. The total number of points used in the analysis varied between the DEMs. This was due to the varying cell size of the respective DEMs. The data were examined for each DEM to verify that only one collection point was within a single cell. As a result, the total number of points available after differential correction for the 10-m, 30-m, and 100-m DEMs were 1125, 1113, and 995 points, respectively.

In this study, raster DEMs were used with square cells.

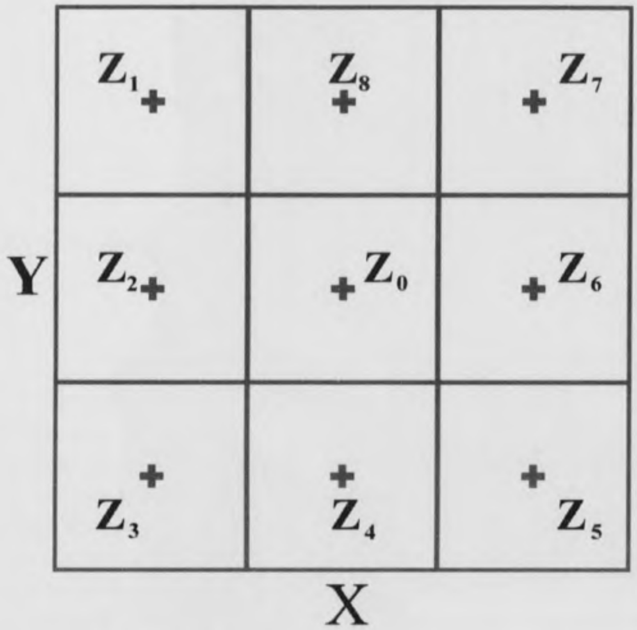

Fig. 2. Notation for elevations used in computing slope from an altitude matrix window.

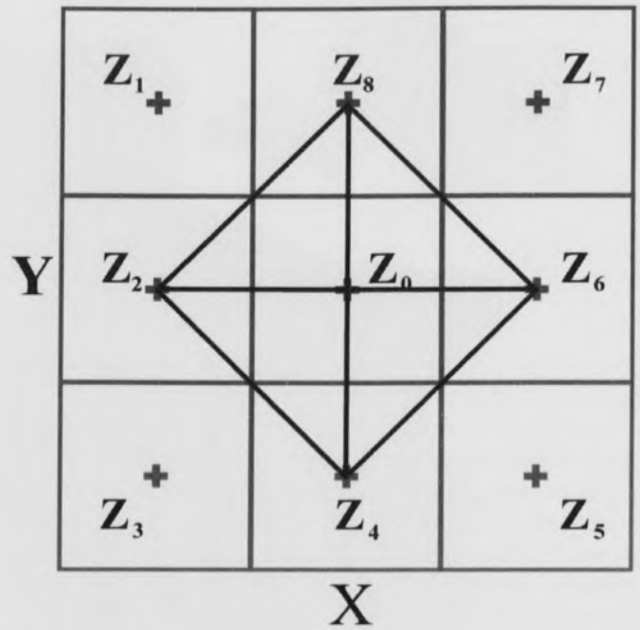

Fig. 3. Configuration of the four right triangles used by method four to determine slope.

The 30-m and 100-m DEMs were obtained from the USGS. Their scales are 1:24,000 and 1:100,000, respectively (Isaacson and Ripple, 1990).

The 10-m DEM was constructed using contour lines digitized from a 1:24,000 topographic map. The accuracy of a DEM is affected by several factors such as the user, digitizer, and the quality of the map. The contour lines were digitized from a Digital Raster Graphic, which is a scanned USGS quadrangle map, with Arcview® 3.x using a method referred to as "heads-up" digitizing. With "heads-up" digitizing, Arcview® 3.x users can zoom in on areas where the contour lines are closer together and more difficult to distinguish, making correction easier. The contour lines were digitized from DRGs that were produced from 1995 to 1998. The elevation values associated with these lines were entered and verified. The quality of the resulting DEM is highly dependent on the accuracy level of the digitized contour data (Robinson, 1994). The overall accuracy of a DRG is approximately the same as the accuracy of the map from which it was derived (USGS, 2004a). According to the National Map Accuracy Standards, the horizontal accuracy of a 1:24,000 topographic map must be within 40 feet (12.2 meters), which is 0.02 inches on the map. The vertical accuracy must be within half of the contour interval (USGS, 2004b). The contour interval of the topographic map used for this study was 20 feet, and therefore the vertical accuracy is within 10 feet.

These digitized contour lines were used to create a 10-m DEM in ArcInfo® using the topogrid command (ESRI©, 2002). In addition to the digitized contour lines, the digitized streams were also needed to use the topogrid command. The streams were digitized in the direction of the flow from the DRG. Eklundh and Martensson (1995) concluded that contour lines should only be used as input data if a sophisticated interpolation algorithm, such as the spline method, is used. The topogrid command is a spline interpolation method specifically designed for the creation of DEMs from comparatively small elevation and stream coverage's (ESRI©, 2002). Once the DEM was created, it was visually examined in ArcView® 3.x for errors.

Eight methods were used to calculate slope from the three DEMs. These are referred to numerically since no other formal names exist. These methods represent a variety of mathematical approaches to calculating slope.

Weih (1991) explained that the equation $z = f(x, y)$ describes points on a three dimensional terrain surface with z equal to the perpendicular distance from the terrain surface point P (x, y) lying on a plane referenced by X and Y coordinates. For the following equations the west-east coordinate direction will be denoted by X, south-north direction by Y, and the elevation by Z. The delta (δ) notation is used to represent the difference in the X, Y, and Z-axes. Equation (1) can be used to calculate slope angle (ϕ) in radians.

$$\tan\phi = \sqrt{\left(\frac{\delta Z}{\delta X}\right)^2 + \left(\frac{\delta Z}{\delta Y}\right)^2} \qquad (1)$$

For the methods used in this study, slope is determined for each cell using elevations from a 3 X 3 cell grid (Fig. 2). Slope values were recorded in percent. There are different ways to interpret the elevation values of the cells that comprise a grid. The elevation value stored can be thought of as the elevation for every point within the cell or the elevation of the center of the cell (Van Krevald, 1997). All of the slope methods in this study treat the elevation value of a cell as a point (cell centroid), even though the value applies to the area contained by each cell in the respective DEM.

Slope Method 1: This is the most common method for calculating slope. The following steps are used to determine the slope of cell Z_0 (Horn, 1981):

$$[\delta Z / \delta X] = [Z_2 - Z_6] / 2^*\Delta X \text{ and} \qquad (2)$$

$$[\delta Z / \delta Y] = [Z_4 - Z_8] / 2^*\Delta Y. \qquad (3)$$

Where
ΔX is the spacing between points in the horizontal direction and
ΔY is the spacing between points in the vertical direction
The slope of the cell Z_0 can be determined by substituting equations (2) and (3) into equation (1).

Slope Method 2: There are two steps involved in determining the slope of Z_0 using this method.

$$[\delta Z / \delta X] = [Z_0 - Z_6] / \Delta X \text{ and} \qquad (4)$$

$$[\delta Z / \delta Y] = [Z_0 - Z_8] / \Delta Y. \qquad (5)$$

The slope of cell Z_0 can be determined with the appropriate substitutions into equation (1) (O'Neill and Mark, 1985, 1987).

Slope Method 3: This method is described in Travis et al. (1975). Slope is determined for Z_0 by calculating the slope from Z_0 to each of its eight neighbors by taking the absolute value of the difference in elevation between Z_0 and each of its neighbors divided by cell size. The maximum slope of the eight calculated slopes is then assigned to cell Z_0.

Slope Method 4: This method is called the plane algorithm method (Struve, 1977). The plane algorithm method calculates the slope of the four surrounding right triangle planes that have Z_0 as a common point. The maximum slope calculated is assigned to cell Z_0. The slope for each plane is calculated similar to method two. Fig. 3 shows the four planes that are used to determine the slope value assigned to Z_0.

Slope Method 5: Struve (1977) used this method in which the eight neighbors of cell Z_0 are used to calculate the maximum slope of two three-dimensional surfaces, S and S'. Surface S uses the four nearest neighbors, which are Z_2, Z_4, Z_6, and Z_8, to determine the partial derivative for the X and Y directions. The next nearest neighbors Z_1, Z_3, Z_5, and Z_7 are used to determine the partial derivative for the X' and Y' directions of surface S'.

$$[\delta Z / \delta X] = [Z_2 - Z_6] / 2^*\Delta X \qquad (6)$$

$$[\delta Z / \delta Y] = [Z_4 - Z_8] / 2^*\Delta Y \qquad (7)$$

$$[\delta Z / \delta X'] = [Z_3 - Z_7] / 2^*\sqrt{2}^*\Delta X \qquad (8)$$

$$[\delta Z / \delta Y'] = [Z_5 - Z_1] / 2^*\sqrt{2}^*\Delta Y \qquad (9)$$

The maximum absolute value derived from the partial derivatives in each direction is substituted in equation (1) to calculate slope. The partial derivatives do not necessarily have to come from the same surface.

Slope Method 6: Horn (1981) proposed a third-order infinite method for calculating slope. This method uses a weighting of three central differences.

$$[\delta Z / \delta X] = [(Z_1 + 2Z_2 + Z_3) - (Z_7 + 2Z_6 + Z_5)] / 8^*\Delta X \text{ and} \qquad (10)$$

$$[\delta Z / \delta Y] = [(Z_1 + 2Z_8 + Z_7) - (Z_3 + 2Z_4 + Z_5)] / 8^*\Delta Y. \qquad (11)$$

Substituting the results of the above equations into equation (1), the slope value is calculated for Z_0. ArcView® 3.x and ArcInfo® use this method to calculate slope.

Slope Method 7: This method uses a third-order finite difference model for calculating the slope of Z_0. This method is similar to method six. The only difference is a change in the weighting of cells Z_2, Z_4, Z_6, and Z_8. Using the Sharpnack and Akin (1969) slope model, equations (10) and (11) can be rewritten as follows:

$$[\delta Z / \delta X] = [(Z_1 + Z_2 + Z_3) - (Z_7 + Z_6 + Z_5)] / 6^*\Delta X \text{ and} \qquad (12)$$

$$[\delta Z / \delta Y] = [(Z_1 + Z_8 + Z_7) - (Z_3 + Z_4 + Z_5)] / 6^*\Delta Y. \qquad (13)$$

The slope value is calculated for Z_0 by substituting the results of the above equations into equation (1).

Slope Method 8: This method, used in Travis et al. (1975), uses a multiple linear regression model. A surface is fitted to a 3 x 3 cell window using least squares. The least squares method minimizes the squared difference between the fitted surface and the cell elevations. The regression model is

$$Z_i = \beta_0 + \beta_1 X_i + \beta_2 Y_i + \varepsilon_i \qquad (14)$$

Assuming that ε_ι is approximately uncorrelated with X and Y, the partial derivatives with respect to X and Y are shown in equations (15) and (16). Substituting equations (15) and (16) into equation (1), the slope value for cell Z_0 can be obtained.

$$\text{Substituting } [(\partial E(Z) / \partial X] = \beta_1 \text{ for } (\delta Z / \delta X) \qquad (15)$$

$$\text{Substituting } [(\partial E(Z) / \partial Y] = \beta_2 \text{ for } (\delta Z / \delta Y) \qquad (16)$$

This method uses all nine elevation values to fit the surface and estimate the slope of cell Z_0.

In order to calculate slope using these eight methods, C++ programming was used. ArcView® 3.x Spatial Analyst has a C programming application program interface (API). This API is a grid data set input/output library that allows the user to read and write data to and from ESRI© grids (ESRI©, 1999). These grid data sets were then viewed and analyzed in ArcView® 3.x.

There were two phases of analysis in this study. The first phase was a cell-by-cell comparison of all three DEMs to determine if there was a relationship between the calculated slope values for the eight different calculation methods. The second phase was to determine the method that calculated the most accurate slope from a 10-m, 30-m, and 100-m DEM by comparison with field slope measurements.

A two-sided paired t-test was performed using the statistical program SAS® to determine if the mean difference between the calculated and measured slopes for a particular DEM was statistically significant (H_o: $\mu_d = 0$ and H_1: $\mu_d \neq 0$, where μ_d = the mean difference between the measured and calculated slope). An alpha (α) level of 0.05 was used for this test. The eight slope methods tested in phase one were tested in phase two.

Results and Discussion

Phase One.--The first phase was a cell-by-cell comparison of all three DEMs to determine if there was a relationship between the calculated slope values for the eight different calculation methods. For the 10-m, 30-m, and 100-m DEMs 1,116,896, 146,914 and 11,645 cells were compared, respectively. All the slope methods were found to be statistically different. These results could be due to the large sample size. For this reason, it is more useful to compare the methods using mean differences and variances. The absolute differences are used for comparison with a negative number showing overestimation and a positive number showing underestimation.

Table 1 shows the slope method differences for the 10-m DEM. For the 10-m DEM, twelve of the slope method comparisons, 1-2, 1-6, 1-7, 1-8, 2-6, 2-7, 2-8, 3-5, 4-5, 6-7, 6-8, and 7-8, had a mean difference of less than +/- one percent. For practical applications, these methods can be considered the same.

Table 2 shows the slope method differences for the 30-m DEM. For this DEM, thirteen of the comparisons, 1-2, 1-6, 1-7, 1-8, 2-6, 2-7, 2-8, 3-4, 3-5, 4-5, 6-7, 6-8, and 7-8, had a mean difference of less than +/- one percent. Twelve of the comparisons were the same as found using the 10-m DEM. As with the 10-m DEM, these methods can be considered the same for most applications. Overall, for the 30-m DEM, methods 7 and 8 were the most similar and methods 4 and 8 with a mean difference of 4.2040% were the least similar.

Table 3 shows the slope method differences for the 100-m DEM. A mean difference of less than +/- one percent was found for the same thirteen comparisons as with the 30-m DEM. Overall, for the 100-m DEM, methods 7 and 8 were the most similar and methods 4 and 8 with a mean difference of 1.9542% were the least similar.

Phase Two.--The second phase was to determine the method that calculated the most accurate slope from a 10-m, 30-m, and 100-m DEM by comparison with field slope measurements. A paired t-test (α = 0.05) was used to determine if the mean difference between the measured and calculated slopes was statistically significant. The results are shown in Table 4. For the 10-m DEM, all of the methods were found to be statistically different. Method 2, with a mean difference of 0.71% and a p-value of 0.0137 was the least different. For the 30-m DEM, the mean slope calculated using method 4 with a p-value of 0.5126 was not statistically different from the mean measured slope (Table 4). All the other methods had p-values from < 0.0001 to 0.0157. The calculated slopes using method 3 with a mean difference of 0.90% is for most practical purposes, the same as the measured slope. For the 100-m DEM, all the methods returned p-values of < 0.0001. A comparison of the mean differences in Table 4, which ranged from 9.28% to 11.08%, shows an underestimation of slope using all eight methods. For all the methods, slopes above approximately 40% were always underestimated. This is illustrated in Fig. 4 for slope Method 1 for the 10-m and 30-m DEMs.

The results of the cell-by-cell comparison of this study differed from those found by Weih (1991). Weih performed a cell-by-cell comparison of a 30-m DEM of Wise and Lee Counties in southwestern Virginia. The DEM elevation range for the Weih (1991) study was from 424-1079 m, a difference of 655 m. The elevation range of the 30-m DEM used in this study was 211-577 m, a difference of 366 m. Also, there were half as many cells used in the cell-by-cell comparison in Weih's study, 77,855 as opposed to 146,914. He found less than +/- one percent difference between slope methods 1-6, 1-7, 1-8, 6-7, 6-8, and 7-8. The results of this study show a less than +/- one percent difference between slope methods 1-2, 1-6, 1-7, 1-8, 2-6, 2-7, 2-8, 3-4, 3-5, 4-5, 6-7, 6-8, and 7-8. Weih (1991) found the smallest mean difference, 0.00007, between methods 7 and 8. In this study,

Table 1. Comparisons of Slope Method Differences on a Cell by Cell Basis for 10-m DEM with slope differences of less than one percent in bold.

Mean (Min,Max) Variance	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7	Method 8
Method 1	***-0.4872*** (-38.82, 25) 15.4479	-2.7700 (-32.93, 9.05) 8.5666	-3.7742 (-38.82, 0) 9.4921	-3.7138 (-19.71, 0) 8.7247	***0.1441*** (-7.28, 11.99) 1.6058	***0.1616*** (-9.70, 15.59) 2.8406	***0.1651*** (-9.70, 15.59) 2.8382
Method 2		-2.2828 (-45.57, 14.14) 22.3280	-3.2870 (-50.00, 0) 22.0992	-3.2266 (-30.37, 29.93) 24.9513	***0.6313*** (-23.25, 38.68) 17.0471	***0.6402*** (-22.67, 38.71) 18.37	***0.6524*** (-22.67, 37.31) 18.2649
Method 3			-1.0043 (-16.57, 21.21) 6.7685	***-0.9438*** (-22.46, 28.27) 15.6069	2.9141 (-5.37, 30.99) 7.1025	2.9095 (-5.88, 33.99) 7.4462	2.9351 (-5.88, 32.93) 7.2806
Method 4				***0.0604*** (-19.71, 36.30) 19.0652	3.9183 (-5.38, 47.05) 11.4194	3.9412 (-7.20, 49.50) 12.72	3.9394 (-7.20, 49.50) 12.7127
Method 5					3.8579 (0, 20.34) 7.2891	4.0135 (0, 21.59) 7.90	3.8789 (0, 20.68) 7.5667
Method 6						***0.0210*** (-2.43, 3.61) 0.1765	***0.0211*** (-2.43, 3.61) 0.1762
Method 7							***0*** (-0.00001, 0.00001) 0

N = 1,116,896 Difference = Row – Column Units = Percent Slope

Table 2. Comparisons of Slope Method Differences on a Cell by Cell Basis for 30-m DEM with slope differences of less than one percent in bold.

Mean (Min,Max) Variance	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7	Method 8
Method 1	***-0.5800*** (-42.71, 30.55) 21.1880	-3.2420 (-41.33, 6.79) 14.1043	-4.0178 (-42.71, 0) 14.7450	-3.1324 (-21.14, 0) 6.9488	***0.1460*** (-7.69, 7.98) 0.4685	***0.1836*** (-10.36, 10.43) 0.8343	***0.1863*** (-10.36, 10.43) 0.8301
Method 2		-2.6620 (-62.45, 16.74) 30.7894	-3.4378 (-62.49, 0) 30.9947	-2.5525 (-47.11, 41.35) 28.8279	***0.7260*** (-29.75, 45.66) 21.6280	***0.7604*** (-29.74, 45.31) 21.87	***0.7662*** (-29.73, 45.32) 21.9688
Method 3			***-0.7758*** (-19.69, 18.96) 3.1088	***0.1095*** (-21.49, 39.02) 19.0223	3.3880 (-3.80, 41.65) 13.8338	3.4226 (-4.03, 42.33) 13.89	3.4282 (-4.02, 42.33) 13.9330
Method 4				***0.8853*** (-18.27, 41.35) 19.5035	4.1638 (-4.33, 45.66) 15.5009	4.2036 (-5.88, 45.31) 15.88	4.2040 (-5.88, 45.32) 15.9501
Method 5					3.2784 (0, 21.20) 6.7536	3.3407 (0, 21.39) 7.0007	3.3187 (0, 21.40) 6.9186
Method 6						***0.0385*** (-2.67, 2.46) 0.0523	***0.0403*** (-2.67, 2.46) 0.0521
Method 7							***0.0015*** (0, 0.0069) 0

N = 146,914 Difference = Row – Column Units = Percent Slope

Table 3. Comparisons of Slope Method Differences on a Cell by Cell Basis for 100-m DEM with slope differences of less than one percent in bold.

Mean (Min,Max) Variance	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7	Method 8
Method 1	***-0.2550*** (-19.87, 20.62) 5.9034	-1.6840 (-26.39, 7.91) 5.8217	-1.8716 (-32.33, 0) 5.2281	-1.6305 (-14.59, 0) 2.7179	***0.0605*** (-4.88, 5.48) 0.3722	***0.0734*** (-6.73, 7.29) 0.6616	***0.0735*** (-6.73, 7.29) 0.6616
Method 2		-1.4290 (-39.45, 11.29) 9.5993	-1.6166 (-41.64, 0) 8.6397	-1.3755 (-30.98, 17.73) 8.2515	***0.3155*** (-19.95, 22.08) 6.3146	***0.3285*** (-19.80, 23.38) 6.6104	***0.3285*** (-19.80, 23.39) 6.6104
Method 3			***-0.1876*** (-11.29, 16.69) 2.2345	***0.0535*** (-15.98, 22.89) 5.3833	1.7445 (-3.41, 28.19) 5.3685	1.7575 (-2.52, 28.79) 5.3516	1.7576 (-2.52, 28.79) 5.3517
Method 4				***0.2411*** (-12.63, 28.83) 5.2296	1.9321 (-3.23, 34.13) 6.0348	1.9451 (-4.53, 34.73) 6.4438	1.9542 (-4.53, 34.73) 6.4439
Method 5					1.6910 (0, 16.02) 2.6916	1.7040 (0, 16.72) 2.8423	1.7040 (0, 16.72) 2.8425
Method 6						***0.0130*** (-1.84, 1.81) 0.0417	***0.0130*** (-1.84, 1.81) 0.0417
Method 7							***0.00007*** (0, 0.0004) 0

N = 11,645 Difference = Row – Column Units = Percent Slope

the mean difference between these two methods was 0.0015, which was also the smallest mean difference.

As stated previously, Weih (1991) used a 30-m DEM with an elevation range of 655 m, where as the elevation range of the 30-m DEM used in this study was 366 m. It can be concluded that with a greater elevation range there were more steep slopes in Weih's area than this study area. As discussed previously, the eight slope calculation methods tended to underestimate slope for the 30-m DEM, especially those slopes greater than approximately 40%. In this study, methods 3, 4, and 5 had the fewest underestimations, which accounts for the similarity between them. Weih (1991) found mean differences between methods 3-4 and 3-5 of 8.29% and 9.74%, respectively. One possible explanation for this is that the steeper slopes may have been underestimated more by one method versus another in Weih (1991), resulting in more of a difference using these methods. The number of cells used in the analysis may have also been a factor in the difference.

In phase two of the analysis, the 10-m DEM would be expected to calculate the most accurate slope since an elevation value assigned to a 100 m^2 area is more accurate than an elevation value assigned to a 900 m^2 or 10,000 m^2 area. The results showed that all but one of the mean slopes calculated from the three DEMs were statistically different from the mean measured slopes. The one exception was the mean slope calculated from the 30-m DEM using method 4, shown previously in Table 4. However, the mean difference between the measured and calculated slope for the 10-m DEM using method 2 was 0.71%. While the paired t-tests showed these slopes were statistically different, they can be considered the same for most practical purposes. The accuracy of the 10-m DEM could account for the results obtained. While the 10-m DEM was examined for errors, a comparison with actual elevation values was not done. The objective of this study is to determine which method calculates the most accurate slope from each DEM. The results of the 10-m DEM are only applicable when the same method of DEM creation is used. As reported by Weih and Smith (1990), different interpolation methods using the same sample data can produce entirely different DEMs.

The underestimation of the slope calculations from the 100-m DEM was expected. A 100-m DEM is composed of 10,000 m^2 cells. This is a large area in which to apply one elevation value (recorded to the nearest meter). As a result, there tends to be a smoothing of the terrain, which contributes to the lower calculated slopes. These results follow those of Chang and Tsai (1991), who found accuracy of slope decreases with lower DEM resolutions.

The elevation values of a USGS DEM, such as the 30-m and 100-m DEMs used in this study, are integer values recorded in meters (Weih and Smith 1990). Consider the

Table 4. Comparisons of Slope Methods for the 10-m, 30-m, and 100-m DEMs with slope differences of less than one percent in bold.

Mean (Min,Max) Variance p-value	10-m DEM	30-m DEM	100-m DEM
Method 1	1.52 (-24.50, 63.00) 74.87 < 0.0001	4.01 (-32.95, 64.01) 95.85 < 0.0001	11.01 (-26.20, 67.82) 148.06 < 0.0001
Method 2	**0.71** **(-32.00, 68.00)** **91.92** **0.0137**	3.28 (-60.31, 63.37) 124.85 < 0.0001	10.75 (-26.67, 67.11) 155.81 < 0.0001
Method 3	-1.55 (-32.00, 58.00) 75.88 < 0.0001	**0.90** **(-60.21, 62.96)** **108.70** **0.0040**	9.44 (-33.70, 64.78) 154.87 < 0.0001
Method 4	-2.63 (-33.23, 58.00) 82.45 < 0.0001	**0.21** **(-60.31, 62.96)** **111.88** **0.5126**	9.28 (-34.22, 66.63) 162.72 < 0.0001
Method 5	-2.01 (-29.62, 61.88) 83.05 < 0.0001	1.06 (-36.10, 62.65) 109.01 0.0007	9.57 (-32.99, 66.03) 158.83 < 0.0001
Method 6	1.69 (-23.04, 63.00) 71.72 < 0.0001	4.142 (-32.95, 63.54) 94.75 < 0.0001	11.07 (-23.15, 67.08) 145.89 < 0.0001
Method 7	1.82 (-20.92, 62.73) 70.56 < 0.0001	4.18 (-32.95, 63.36) 94.53 < 0.0001	11.08 (-22.15, 66.83) 145.28 < 0.0001
Method 8	1.72 (-22.57, 63.00) 71.41 < 0.0001	4.18 (-32.94, 63.36) 94.53 < 0.0001	11.08 (-22.15, 66.83) 145.28 < 0.0001

scenario where the following elevation values are assigned to a 3 X 3 neighborhood of a 30-m DEM: Z_1 through $Z_7 = 10$ and $Z_8 = 11$. The calculated slope using method 3 would equal 3.56%. If $Z_8 = 12$ instead of 11, then the calculated slope would equal 7.13%. Since slope values can only vary by integer values in meters, it is not possible to get a slope value between 3.56% and 7.13% using slope method 3. Since the 10-m DEM used in this study was not obtained from USGS, its original elevation values were not integers.

Summary

In this study, method 2 calculated the most accurate slope for the 10-m DEM. Method 4 calculated the most accurate slope for the 30-m DEM. However, method 3, although statistically different from the measured slope, can be considered practically equal to it. All methods greatly underestimated slope for the 100-m DEM with methods 3, 4, and 5 having the smallest mean slope differences of 9.28%, 9.44%, and 9.57%, respectively. When considering error, slope values calculated from the 30-m DEM using method 4 were the most accurate. As when error was not considered, all methods greatly underestimated slope for the 100-m DEM, but methods 3, 4, and 5 had the smallest mean differences of 8.84%, 8.71%, and 9.03%, respectively. For all methods, steeper slopes, greater than approximately 40%, were underestimated.

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