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Prioritizing Interdictions on a Shortest Path Network

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Abstract

We consider a variant to the shortest path network interdiction problem with symmetric information from Israeli and Wood (Networks 40, 97-111,2002) which arises in the context of nuclear smuggling prevention. In the basic shortest path interdiction problem, an interdictor has a limited number of interdictions with which he can lengthen arcs in a network in order to maximize the length of the network's shortest path. This thesis considers the case in which the interdictor does not make all of the interdictions at once. Rather, the interdictor must make the interdictions over a set number of periods. Each period has a budget for the number of interdictions that can be placed during the period. The interdictor must prioritize the interdictions and decide the order in which the interdictions should take place. This problem is formulated as an integer program with an objective to maximize the average of the shortest paths across all periods.

Table of Contents

List of Tables

List of Figures

Section 1- Introduction

Network interdiction as described by Collado and Papp [2] is the "monitoring or halting of an adversary's activity on a network." The adversary maneuvers across the network with the intent to optimize an objective such as minimizing the likelihood of being detected, minimizing the travel distance across the network, or maximizing the flow capacity for goods through the network. An interdictor affects the network in order to combat the adversary.

The shortest path network interdiction problem models a hostile situation in which two forces, an interdictor and an attacker, compete. The attacker desires to find the shortest path on a network from an initial node to the final node. The interdictor's role is to eliminate or lengthen arcs in the network to maximize the shortest path the attacker can take. It is assumed that the attacker is aware of the interdictor's impact on the network, so the situation is a Stackelberg game, a concept first described by economist Heinrich Stackelberg in his 1934 publication, Market Structure and Equilibrium [9]. The shortest path maximization problem is described in further detail by Israeli and Wood [4] but we summarize the relevant concepts in this paper.

Network interdiction problems have various real world applications. One of the main applications discussed in literature is nuclear smuggling. Morton et al. [7], Michalopoulos et al. [5], and others use this application as the focus of their discussion and modeling of network interdiction. In this case, the arcs of the network represent the checkpoints along and surrounding the border of a country. Probabilities are associated with each arc that describe the attacker's chance of smuggling nuclear material through the checkpoint undetected. The interdictor has a limited number of detectors he can place to increase the probability of detecting the nuclear material at a given checkpoint. Other applications include infectious disease control, counterterrorism, and smuggling of other sorts as discussed by Collado and Papp [2]. The maximum-reliability path has received attention on a general network as seen in Dimitrov's work in nuclear smuggling [3]. The maximum-reliability path interdiction problem can be transformed into a shortest path interdiction problem by applying a logarithm to the objective as shown by Morton [6].

There are several variants and extensions to the network interdiction problems. One variant is the asymmetric information case where the attacker and interdictor have different levels of information about the network [1]. As demonstrated by Salmeron [8], asymmetric interdiction models can be used when the interdictor has the possibility to exploit an information advantage by using deception tactics. Another variant is stochastic network interdiction. As addressed by Morton et al. [7], in a stochastic network interdiction the attacker's characteristics are not known with certainty. The prioritization variant, which is overviewed in the following paragraph, is considered in this research. Michalopoulos et al. [5] models this case on a bipartite network. This thesis models prioritization on a general network.

In the prioritization case the interdictor does not make all of the interdictions at once. Rather, the interdictor must make the interdictions over a set number of periods. Each period has a budget for the number of interdictions that can be placed during the period. The interdictor must prioritize the interdictions and decide the order in which the interdictions should take place. This case can be applied to the nuclear smuggling application. A country attempting to secure its borders may only have the budget to purchase and place detectors periodically. Rather than placing the interdictions all at once and maximizing the shortest path or probability of detection, the nation may have a yearly budget for detectors and must place them in sequence. Because the placement of detectors occurs over time, there is a non-negligible possibility that smuggling occurs before detector placement is complete. This study attempts to model a prioritization scenario and decide the optimal allocation of interdictions that maximizes the average shortest path length through a network across all time periods.

The formulation of the model as an integer program that solves the prioritization network interdiction problem on a general network is developed in Section 2. Section 3 describes the generation of test networks and the data used to test the proposed model. The results from these test networks are analyzed in Section 4, and conclusions and recommendations for future work are discussed in Section 5.

Section 2 - Methodology

In this section the mathematical formulation of the prioritization problem is described. When solved using optimization software, this integer program will determine for a general network which arcs should be interdicted and in what order they should be interdicted to maximize the average length of the shortest path across the network.

Because determining the shortest path of a network is a minimization problem, the model for maximizing the shortest path length is a classic max-min problem. To model a max-min problem, a dual problem can be created from the shortest path minimization problem. The dual of a minimization problem is a maximization problem. This dual thus creates a max-max problem which can be solved using standard integer programming methods.

In modeling the prioritization problem certain assumptions must be made. First, it is assumed that the interdictor and attacker have identical perceptions. There is symmetric information such that both are aware of the same network. Both are aware of the current arc lengths and what the length of an arc will be if it is interdicted. Next, it is assumed that the attacker can travel both ways on an arc; i.e., the attacker can travel from node 2 to node 3 on an arc (2,3) or from node 3 to node 2. It is also assumed that when an interdiction is placed it affects the travel between the nodes both ways. The objective of the model is to maximize the average of the shortest path across all periods. The model formulated here does not assume extra weight should be attached to making the shortest path longer in earlier periods or having the longest shortest path possible at the end of the last period.

Let $G = (N, A)$ define a directed network with nodes $N = \{1, \dots, s\}$ and arcs $A \subseteq N \times N$. In keeping with the assumption of bi-directional travel on arcs, the directed arc set is assumed to be such that whenever (i, j) is included in A, (j, i) is also included in A. For each arc $(i, j) \in A$ we define $c_{i,j}$ as the current arc length and $w_{i,j}$ as the change in the arc length if an interdiction is placed on the arc. A maximum of b interdictions can be placed in each period, and the total number of periods is f. The set $K = \{1, \dots, f\}$ defines the periods. Binary variable $z_{i,j,k}$ is equal to one if arc (i, j) is interdicted in period k. Let function $g_k(z)$ denote the shortest path length in period k.

An intuitive formulation of the prioritization problem is given by Model (1) – (5).

$$\text{Max} \quad \sum_{k \in K} \frac{1}{f} g_k(z) \tag{1}$$

$$\text{s.t.} \quad \sum_{(i,j) \in A} z_{i,j,k} \leq 2b, \qquad \forall k \in K \tag{2}$$

$$\sum_{k \in K} z_{i,j,k} \leq 1, \qquad \forall (i,j) \in A \tag{3}$$

$$z_{i,j,k} = z_{j,i,k}, \qquad \forall (i,j) \in A, k \in K \tag{4}$$

$$z_{i,j,k} \in \{0,1\}, \qquad \forall (i,j) \in A, k \in K \tag{5}$$

Objective (1) seeks to maximize the average length of the shortest path $g_k(z)$ over all periods. An optimal solution to this problem will provide a sequence of interdictions where order matters in maximizing the average shortest path. For each period k, Constraint (2) limits the number of interdictions to be less than or equal to the number of interdictions available in that period. The budget is multiplied by two because an interdiction placed on an arc (i,j)—as enforced by Constraint (4) —implies the opposing arc (j,i) is also affected by the interdiction. For example, if the arc between node 2 and node 3 is to be interdicted in period 1 so that $z_{2,3,1}$ = 1, travel along the arc is affected from node 3 to node 2 as well so $z_{2,3,1}$ = 1. Constraint (3) ensures that an arc can only be interdicted once. As will become clear when $g_k(z)$ is formally defined in Model (6) – (8), an arc that is interdicted in period k remains at its maximum length throughout the remainder of the time horizon. Constraint (5) prohibits partial interdiction of an arc.

Given fixed interdiction variables z, the shortest path length $g_k(z)$ in period k is defined formally in Model (6) – (8).

$$g_k(z) = \text{Min} \quad \sum_{(i,j)\in A} \left(c_{i,j} + w_{i,j} \sum_{k'=1}^{k} z_{i,j,k'} \right) x_{i,j} \tag{6}$$

$$\text{s.t.} \quad \sum_{j:(i,j)\in A} x_{i,j} - \sum_{j:(i,j)\in A} x_{j,i} = \begin{cases} 1 & i = 1 \\ 0 & i = 2, \dots, s-1 \\ -1 & i = s \end{cases} \qquad : y_{i,k} \tag{7}$$

$$x_{i,j} \geq 0, \qquad \forall (i,j) \in A \tag{8}$$

The variable $x_{i,j}$ is equal to one if the attacker traverses arc (i,j). Objective (6) seeks to minimize the length of the path traversed by the attacker. The length of arc (i,j) is equal to the original length of the arc $c_{i,j}$ plus an additional $w_{i,j}$ if the arc has been interdicted in or before period k. Constraint (7) is the flow balance constraint, which ensures that the path represented by x begins at node 1 and ends at node s. Constraint (8) requires that the x-variables satisfy only nonnegativity; however, the existence of a binary optimal solution to the shortest path linear program is well known.

To convert the max-min problem presented in Model (1) – (5) to a max-max problem the dual of Model (6) - (8) must be taken. Let $y_{i,k}$ denote the dual variable associated the ith Constraint (7). By strong duality, the dual of Model (6) – (8) will have an optimal objective value equal to the optimal objective value in Model (9) – (11), so the dual's optimal objective is equal to $g_k(z)$. The dual is stated in Model (9) – (11).

$$g_k(z) = \text{Max} \quad y_{1,k} - y_{s,k} \tag{9}$$

$$\text{s.t.} \quad y_{i,k} - y_{j,k} \leq c_{i,j} + w_{i,j} \sum_{k'=1}^{k} z_{i,j,k'}\,, \qquad \forall (i,j) \in A, k \in K \tag{10}$$

$$y_{i,k} \text{ unrestricted}, \qquad \forall i \in N \tag{11}$$

Replacing $g_k(z)$ in Model (1) – (5) with the objective of Model (9) – (11) and adding the constraints from Model (9) – (11) to Model (1) – (5) yields the a single-stage (i.e., max-max) formulation of the prioritization problem. This is shown below in Model (12) – (18).

$$\text{Max} \quad \sum_{i \in K} \frac{1}{f} (y_{1,k} - y_{s,k}) \tag{12}$$

$$\text{s.t.} \quad \sum_{(i,j) \in A} z_{i,j,k} \leq 2b, \qquad \forall k \in K \tag{13}$$

$$\sum_{k \in K} z_{i,j,k} \leq 1, \qquad \forall (i,j) \in A \tag{14}$$

$$y_{i,k} - y_{j,k} \leq c_{i,j} + w_{i,j} \sum_{k'=1}^{k} z_{i,j,k'}, \qquad \forall (i,j) \in A, k \in K \tag{15}$$

$$z_{i,j,k} = z_{j,i,k}, \qquad \forall (i,j) \in A, k \in K \tag{16}$$

$$z_{i,j,k} \in \{0,1\}, \qquad \forall (i,j) \in A, k \in K \tag{17}$$

$$y_{i,k} \text{ unrestricted}, \qquad \forall i \in N, k \in K \tag{18}$$

To demonstrate a solution using this model let us compare our approach to another approach on a sample network. Another approach that could be taken to the prioritization problem is the "greedy approach." Rather than looking at the effect on future periods of placing an interdiction as in our model, the greedy approach places an interdiction that will maximize the current shortest path. Consider the following network as an example with two periods having a budget of one interdiction each. The arc lengths $c_{i,j}$ are labeled on each arc (i,j) with the change in length $w_{i,j}$ in parentheses beside the arc lengths.

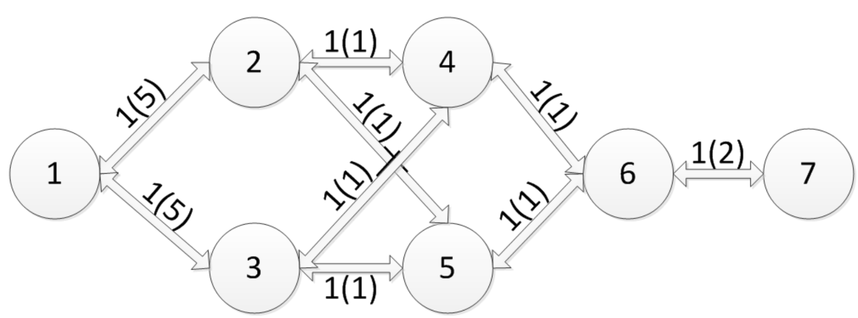

Figure 1 Greedy vs. Prioritization Approach

The greedy approach to the prioritization problem would interdict the arc (6,7) first making the first period's shortest path have a distance of six. In the second period, no interdiction made could extend the shortest path farther, so the shortest path length would be six at the end of the second period as well. The average shortest path length over two periods for the greedy approach is six. Our prioritization model would consider all of the periods at once and would provide the optimal solution of interdicting the arc (1,2) and then the arc between (1,3). The shortest path in the first period would only be four, but at the end of the second period the shortest path would be 9. The average would be 6.5, a better solution than that given by the greedy approach.

Section 3 - Data

To test the model formulation, a series of test networks were generated ranging in network size and number of interdiction periods. Grid networks were generated using VBA and Excel. The CPLEX solver 12.6 in AMPL was then used to solve Model (12) – (18) for the different networks. This is discussed in further detail in Section 4.

Though the prioritization problem and network interdiction problems in general are relevant and real-world applicable, there is not a great source of real world data available for the testing of interdiction models for obvious reasons. Border security and smuggling information is sensitive and hard to reach material. This is one potential reason why Bayrak and Bailey [1] and Salmeron [8] perform computational experiments using randomly generated grid networks of varying size. We utilize a similar approach in this paper to generate a set of test instances for Model (12) – (18).

Preliminary tests were first completed to assess the network size limits of the model and solver. Then, networks were generated ranging in size to observe the capabilities of the model and solver. Each size network had square and rectangular networks generated to mitigate the effect of network width, the number of arcs needed to get from the starting node to the final node of a network. For instance, for the 100-node network size, four column by 25 row networks as well as 10 by 10 networks were generated. For each size configuration, what we called "network sets," three different networks were randomly generated. Each network generated was tested with different numbers of periods and budgets. The following table displays the different network instances used:

Table 1 Test Network Sets

Network Set	**Columns**	**Rows**	**Nodes**
A	3	6	20
B	6	3	20
C	4	25	102
D	10	10	102
E	10	50	502
F	20	25	502
G	20	125	2502
H	50	50	2502

Each network has a dummy source and sink node. These nodes are attached to the network with dummy arcs so that the attacker's path can start from any node in the first column of the network and end at any node in the last column. The arc lengths, parameter $c_{i,j}$, were randomly generated using VBA between one and 50. The change in arc length, parameter $w_{i,j}$ was set to 10 for every arc. The following is a six column by three row network example:

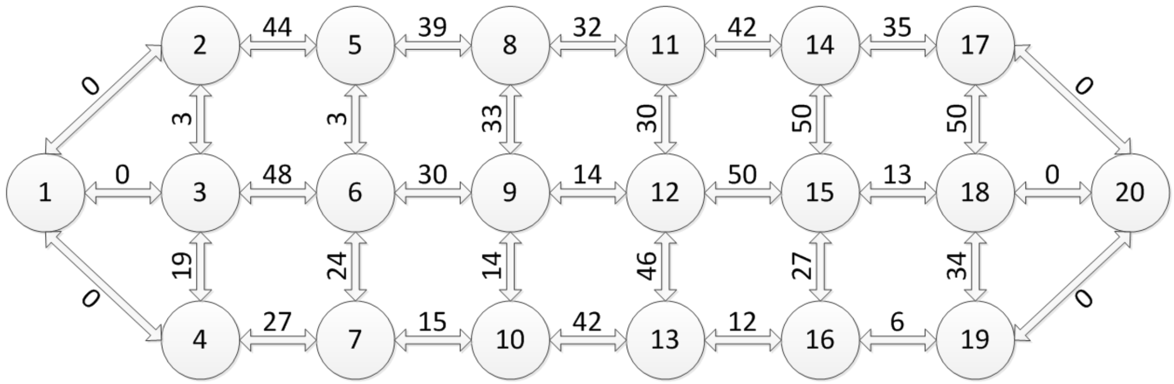

Figure 2 Network Example B1

Section 4 - Results

The solver CPLEX was used to solve the model on each of the test networks generated. This test run we call the "Prioritization Test." An additional round of tests was run on the networks with a different objective. The objective was changed from maximizing the average shortest path length to maximizing $y_{1,f} - y_{s,f}$, the final shortest path length. Under this objective, the interdiction problem reduces to a basic shortest path network interdiction problem, i.e., as modeled by Israeli and Wood [4], with no prioritization. This test run is referred to as the "At Once Test" as all of the interdictions are placed at once as opposed to in sequence. These results can be compared to see how effective our model is at not only maximizing the average shortest path but also at maximizing the final shortest path.

The test runs were limited to 3600 seconds on each of the networks. For each test run the solve time, relative optimality gap percentage, objective value, and final shortest path distance were recorded. The results for each test run are summarized in the table below. Three different networks were tested for each network set. Each network was tested with four different levels of interdictions. Each test was run again with the at once test objective of maximizing the shortest path without periods. The solve time was recorded. The objective value for the prioritization test is the average of the shortest path lengths across all periods. The objective value for the at once test is the maximum shortest path length with all interdictions placed at once. The relative optimality gap percentage (RELMIPGAP) is the percentage difference between the objective value and the objective value of the best bound remaining. The final shortest path length for each prioritization test was recorded to compare to the objective of the at once test for each network. The difference between the two is displayed in the far right column.

Table 2.1 Test Results

Network		**Prioritization Test**				**At Once Test**			
Network	Interdi-ctions	Solve Time (Limit 3600)	Objective	RELMIPGAP	Final SP Length	Solve Time (Limit 3600)	Objective	RELMIPGAP	Test Difference
A1	3	0.11	24.3333	0	31	0.062	31	0	0
A1	5	0.094	27	0	31	0.078	31	0	0
A1	10	0.125	29	0	31	0.109	31	0	0
A1	15	0.172	29.6667	0	31	0.125	31	0	0
A2	3	0.078	26.6667	0	30	0.062	30	0	0
A2	5	0.078	28	0	30	0.078	30	0	0
A2	10	0.141	29	0	30	0.11	30	0	0
A2	15	0.249	29.3333	0	30	0.125	30	0	0
A3	3	0.109	35.3333	0	39	0.093	39	0	0
A3	5	0.125	39.6	0	48	0.156	48	0	0
A3	10	0.187	43.8	3.24E-16	48	0.125	48	0	0
A3	15	0.141	45.2	3.14E-16	48	0.125	48	0	0
B1	3	0.093	122	0	132	0.078	132	0	0
B1	5	0.078	132	0	152	0.078	152	0	0
B1	10	0.125	142	2.00E-16	152	0.109	152	0	0
B1	15	0.156	145.333	3.91E-16	152	0.156	152	0	0
B2	3	0.094	88.6667	0	94	0.078	94	0	0
B2	5	0.171	93.6	0	102	0.14	104	0	2
B2	10	0.983	105.6	1.35E-16	124	0.109	124	0	0
B2	15	0.889	111.733	1.27E-16	124	0.25	124	0	0
B3	3	0.078	82	0	92	0.109	92	0	0
B3	5	0.202	92	0	112	0.078	112	0	0

Table 3.2 Test Results Cont.

Network		Prioritization Test				At Once Test			
Network	Interdi-ctions	Solve Time (Limit 3600)	Objective	RELMIPGAP	Final SP Length	Solve Time (Limit 3600)	Objective	RELMIPGAP	Test Difference
B3	10	0.109	102	0	112	0.11	112	0	0
B3	15	0.156	105.333	1.35E-16	112	0.171	112	0	0
C1	5	0.624	45.8	0	51	1.388	51	0	0
C1	10	68.36	50	9.95E-05	56	3606.9	56	0.018728	0
C1	15	3605.65	52.9333	0.020726	60	3606.45	60	0.033647	0
C1	25	3604.98	57.8	0.031765	68	2.496	68	0	0
C2	5	0.53	41.8	0	45	7.145	45	0	0
C2	10	1213.98	45.9	9.97E-05	52	3607.63	52	0.029054	0
C2	15	3604.26	49.6667	0.02122	59	3604.87	59	0.019379	0
C2	25	3602.31	54.52	0.016704	62	1.654	62	0	0
C3	5	0.359	43.2	1.64E-16	47	3.386	47	0	0
C3	10	28.611	48.8	9.98E-05	56	3606.29	56	0.013616	0
C3	15	1640.77	52.5333	9.99E-05	62	0.686	62	0	0
C3	25	3601.44	56.32	0.00071	62	1.482	62	0	0
D1	5	0.593	126	0	146	0.25	146	0	0
D1	10	2.715	143.2	1.98E-16	170	0.655	170	0	0
D1	15	122.258	154.533	9.96E-05	182	3603.81	182	0.015	0
D1	25	3602.17	169.76	0.009819	200	3602.89	200	0.006478	0
D2	5	0.25	143.2	0	156	0.203	156	0	0
D2	10	0.764	158.1	1.80E-16	183	0.53	183	0	0
D2	15	353.28	168.6	9.89E-05	195	3.322	195	0	0
D2	25	3720.01	183.68	0.008736	215	3604.96	215	0.008907	0
D3	5	0.592	139	0	159	0.187	159	0	0
D3	10	1.124	155.3	1.83E-16	180	2.106	180	0	0
D3	15	143.771	165.733	9.96E-05	191	3.026	191	0	0
D3	25	2560.6	178.76	1.00E-04	199	0.514	199	0	0
E1	5	11.576	146	7.80E-05	152	4.524	152	0	0
E1	10	3601.55	151.1	0.008531	159	3604.32	159	0.005462	0
E1	15	3603.12	154.867	0.013376	164	3605.96	164	0.01429	0
E1	25	3602.19	160.88	0.019964	174	3606.37	175	0.014396	1

Table 4.3 Test Results Cont.

Network		Prioritization Test				At Once Test			
Network	Interdi-ctions	Solve Time (Limit 3600)	Objective	RELMIPGAP	Final SP Length	Solve Time (Limit 3600)	Objective	RELMIPGAP	Test Difference
E2	5	11.732	137	7.01E-05	142	1202.66	142	0	0
E2	10	3602.83	142.9	0.007573	151	3602.97	151	0.006144	0
E2	15	3604.06	147.133	0.01256	160	3600.71	160	0.000271	0
E2	25	3667.24	154.12	0.018828	170	3606.48	170	0.008456	0
E3	5	75.442	119.6	2.94E-05	122	3602.64	122	0.016515	0
E3	10	3602.25	124.5	0.01591	132	3602.92	132	0.016759	0
E3	15	3602.05	128.467	0.021541	140	3606.09	140	0.005764	0
E3	25	3676.94	135.24	0.027966	151	3605.17	151	0.023788	0
F1	5	2.512	317	0	330	4.586	330	0	0
F1	10	3602.19	330.2	0.001191	352	3600.69	352	0.000418	0
F1	15	3673.14	338.733	0.004564	361	3601.8	362	0.002762	1
F1	25	3603.73	350.76	0.008742	375	3813.21	375	0.011953	0
F2	5	1.763	302.2	0	311	3.416	311	0	0
F2	10	22.152	314.7	1.81E-16	336	8.268	336	0	0
F2	15	3670.75	324	0.001834	347	3602.08	347	0.007205	0
F2	25	3603.28	337.08	0.005952	366	3722.37	366	0.005531	0
F3	5	11.59	325.4	9.08E-05	332	3.76	332	0	0
F3	10	3601.61	332.7	0.004027	343	3601.8	343	0.005608	0
F3	15	3601.27	338.733	0.0058	353	3601.77	353	0.006213	0
F3	25	3654.37	348.76	0.007914	372	3733.77	372	0.002688	0
G1	5	158.544	267.6	6.43E-05	272	3685.24	272	0.000402	0
G1	10	3649.58	273.1	0.005952	282	3683.04	282	0.00149	0
G1	15	3602.95	277.667	0.006823	291	3655.02	291	0.011455	0
G1	25	3608.99	285.76	0.010404	303	3621.44	303	0.014753	0
G2	5	3601.55	307.4	0.00163	310	3811.48	310	0.004156	0
G2	10	3660.7	311.2	0.005858	318	3683.95	318	0.010782	0
G2	15	3605.89	314.267	0.008888	324	3655.88	324	0.003164	0
G2	25	3617.01	319.16	0.012557	330	3622.52	330	0.008554	0
G3	5	24.945	282.2	0	291	6.069	291	0	0
G3	10	3685.55	292.7	0.002228	309	3692.98	309	0.005277	0

Table 5.4 Test Results Cont.

Network		Prioritization Test				At Once Test			
Network	Interdi-ctions	Solve Time (Limit 3600)	Objective	RELMIPGAP	Final SP Length	Solve Time (Limit 3600)	Objective	RELMIPGAP	Test Difference
G3	15	3605	299.8	0.003982	318	78.5	319	8.37E-05	1
G3	25	3610.04	309.24	0.007993	329	3619.3	329	0.005276	0
H1	5	40.389	780.8	1.46E-16	800	4.898	800	0	0
H1	10	3605.04	794	0.001138	812	71.105	812	0	0
H1*	15	3606.09	751	1.33E+72	751	3633.98	823	0.002277	72
H1*	25	3612.22	751	1.33E+72	751	3612.03	850	0.000503	99
H2	5	33.837	816.8	0	828	4.961	828	0	0
H2	10	3602.39	825.4	0.001099	838	3668.18	838	0.005967	0
H2*	15	3607.13	800	1.25E+72	800	3632.87	848	0.002358	48
H2*	25	3618.52	800	1.25E+72	800	3622.67	868	0.004147	68
H3	5	9.797	724	0	744	5.335	744	1.53E-16	0
H3	10	3033.13	749	1.52E-16	794	14.196	794	0	0
H3*	15	3604.53	694	1.44E+72	694	24.523	824	1.38E-16	130
H3*	25	3611.25	694	1.44E+72	694	147.749	854	0	160

The test results show that the prioritization problem is able to be solved in most cases at or close to the optimal objective value. The relative optimality gap is less than 1% for 80% of the prioritization tests. The gap is less than 3.5% for all but six of the prioritization tests. These tests, noted by the asterisks in the table, did not have a solution reached within an hour. These large networks with higher number of periods push the limit of the solver. As more interdictions are introduced the number of feasible solutions increases exponentially. Also, it should be noted that this occurs in the 50 by 50 networks but not in the 20 by 125 networks. Both sets have networks with 2500 nodes, but the wider networks are not solved as efficiently.

The final shortest path length does not always increase as more interdictions are used. In network A1 for instance, the final shortest path length remains at 31 even when more interdictions and periods are made available. These are cases when no more interdictions can be made to change the attacker's path choice.

The prioritization tests often have solutions with maximum final shortest path lengths as shown in the table to the right. Out of the 90 solved tests, 86 tests resulted in solutions with the maximum final shortest path length. This could show that the solution to the prioritization problem solution often has the same interdictions as made to maximize the final shortest path. It could also show that in some instances the prioritization solution includes a set of interdictions that is completely different from the interdictions made to maximize the shortest path. Further research can be done to determine the relationship between the prioritization problem and at once problem. The results indicate that the prioritization model is highly effective at maximizing the final shortest path length.

An example solution is broken down in the following figures. The network displayed here is the second "B" network generated with five periods of interdictions, B2.5. The network before any interdictions are placed is displayed first. The path highlighted in red is the shortest path. A picture of the network at the end of each period is then displayed with the interdiction placed during that period as well as the new shortest path at the end of that period.

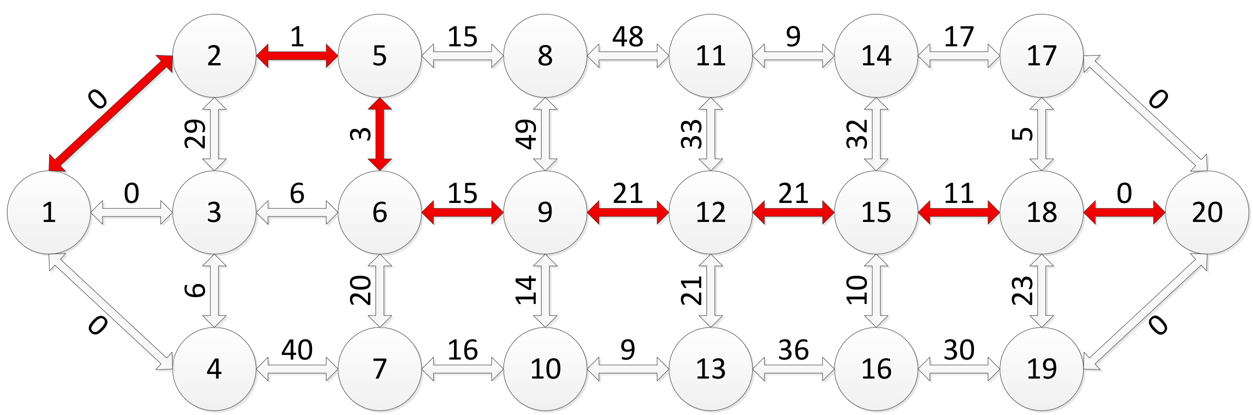

Figure 3 Network B2.5. The shortest path length is 72.

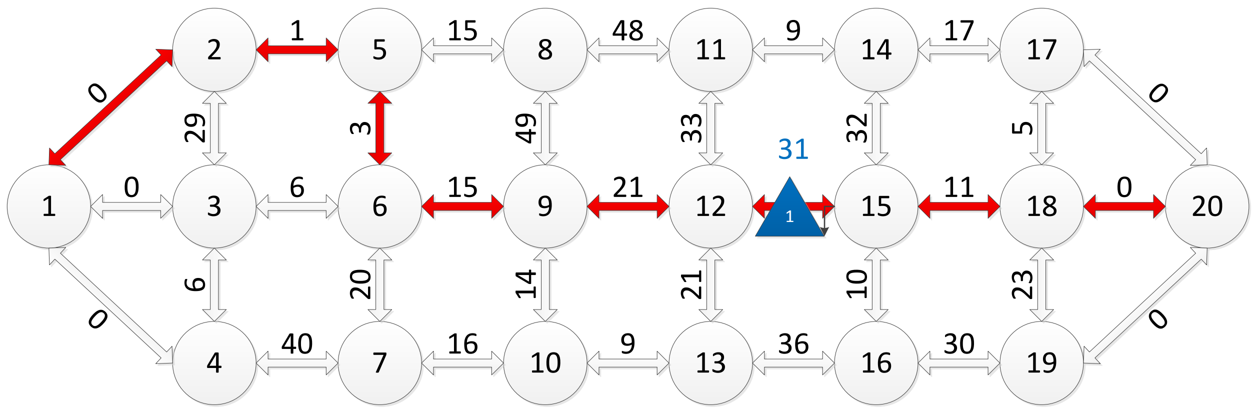

Figure 4 Network B2.5 period 1. The shortest path length is now 82.

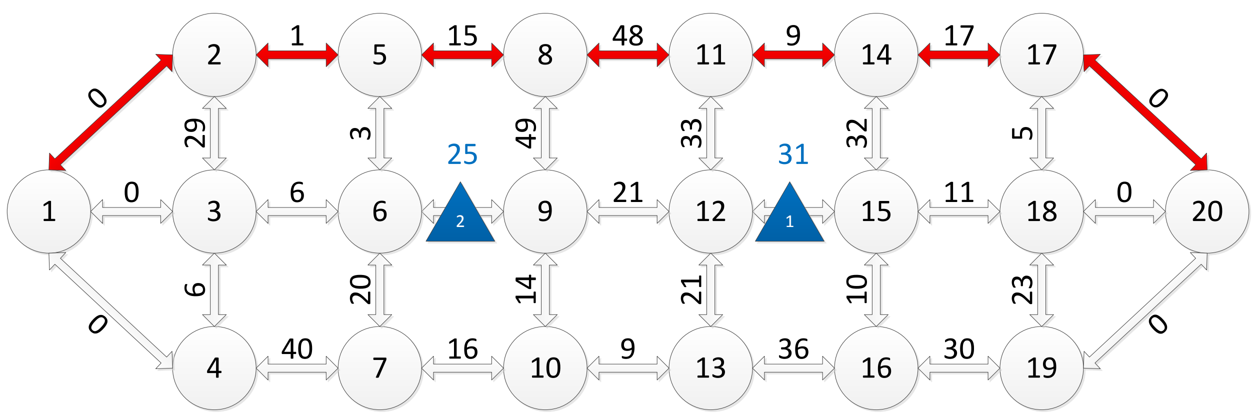

Figure 5 Network B2.5 period 2. The shortest path length is now 90.

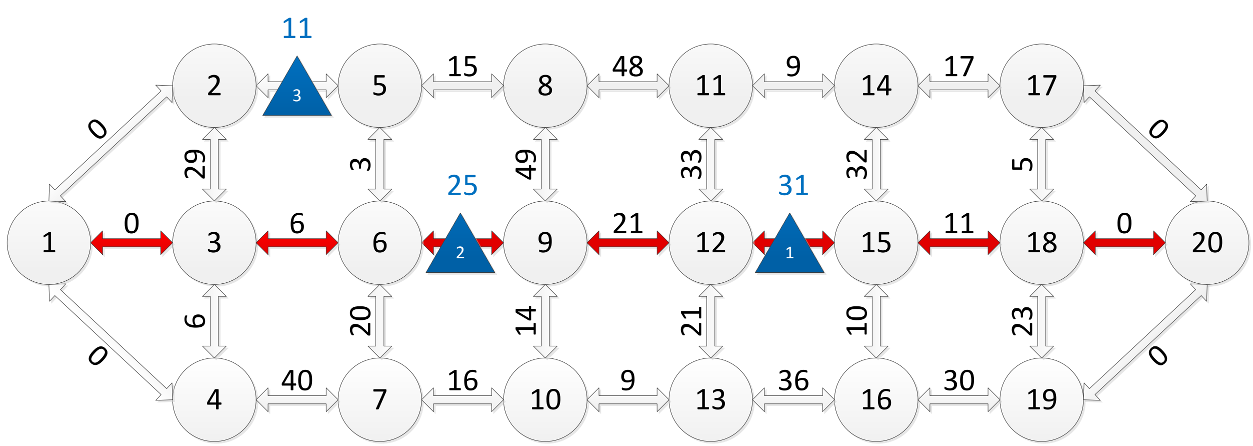

Figure 6 Network B2.5 period 3. The shortest path length is now 94.

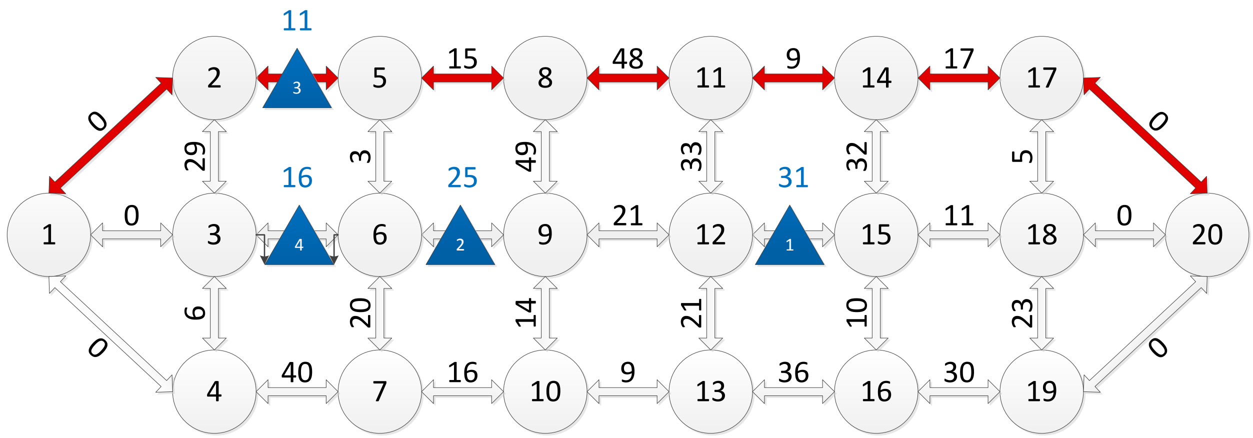

Figure 7 Network B2.5 period 4. The shortest path length is now 100.

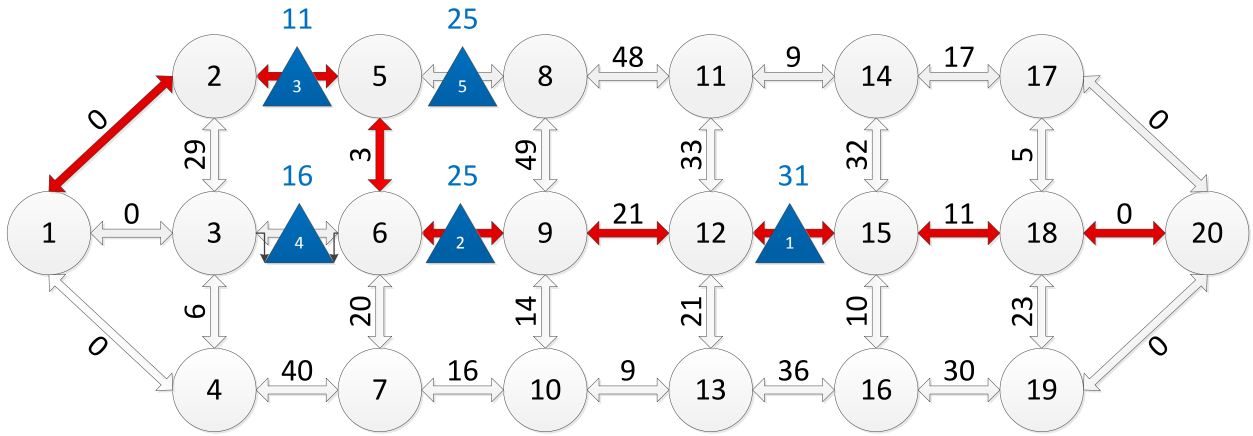

Figure 8 Network B2.5 period 5. The shortest path length is now 102.

The shortest path length at the end of period 5 is 102. The average shortest path length over all periods is 93.6. This is an example of a problem that has alternate optimal solutions. Another solution that the solver did not choose has an average shortest path length of 93.6 and a final shortest path length of 104. This solution has smaller shortest path lengths in the earlier periods. This highlights how no weight is given to the final period and is an example of a case where the solution given by the prioritization model does not maximize the final shortest path length.

Section 5 – Conclusion and Future Research

This paper has designed a model for prioritizing network interdictions on a general symmetric information network under certainty. The solution to the prioritization problem gives the optimal sequence of interdictions that maximizes the average length of the shortest path over a set number of periods. Twenty-four grid networks of varying size were generated to test the model. Another test run, the "At Once" test, was made to evaluate the prioritization model's ability to maximize the shortest path.

The tests run using the prioritization model on a variety of networks and interdiction levels show the model is efficient at solving problems up to networks with 2500 nodes and 25 periods of interdictions. The model also is effective at maximizing the final shortest path of a network. Further research can be done to discover the relationship between the solutions to the prioritization problem and the maximum shortest path "At Once" problem.

The model developed here can pave the way to further research in the network interdiction field. First, using this model as groundwork, different more efficient ways of solving the problem can be innovated. The model has limits when put through a solver, and these limits can be expanded with further work. The solve time could also be shortened making the approach scalable to realistically sized problem instances.

Another area of further research for the prioritization problem could be combining the prioritization variant with other variants of the network interdiction problem. The asymmetric information problem [1] could be solved as a prioritization problem. The deception problem [8] could be made into a prioritization problem as well. Also, the prioritization could be applied to a flow capacity network problem as opposed to the shortest path problem. Stochastic attacker behavior could be modeled as in [6,8] by having an unknown source and sink node.

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