5-2014

Uncertainty in relief supply distribution

Christopher A. Bayles
University of Arkansas, Fayetteville

This thesis is approved.

Thesis Advisor:

Ashlea Munn

Thesis Committee:

Sammy

Uncertainty In Relief Supply Distribution

An Undergraduate Honors College Thesis

in the

Department of Industrial Engineering
College of Engineering
University of Arkansas
Fayetteville, AR

by

Christopher A. Bayles

Uncertainty In Relief Supply Distribution

Chris Bayles

Abstract

In the aftermath of a natural disaster the efficient planning of humanitarian relief efforts, e.g., delivering supplies to locations in need, can often mean the difference between lives being saved and lives being lost. In this paper we focus on two things: (1) appropriate objectives for routing problems in the context of delivery of relief supplies, and (2) how to handle the uncertainty associated with delivery of relief supplies after a disaster. Through applying uncertainty to parameters such as travel time between locations, we seek to examine if/how optimal routes change as a result of different objectives and the presence of uncertainty. After discussing various different parameters and examining the effects of uncertainty in these cases, we then offer up an algorithmic process that could be used in the formation of these routes using the uncertain information. Finally, we analyze the results of a simulation model that ran different routes using information known with certainty and uncertain information, and see what light it sheds on the disaster response discussion.

1   Introduction

Natural disasters are something that people from all over the world are familiar with as no country, developed or developing, have control over these forces of nature. The people of nations affected by a natural disaster, often are in need of essential survival items (food, water, etc) as the destruction left in the disaster's wake frequently disrupts the normal means by which they would acquire these items. Because Mother Nature is not biased based on city size, the population in need of supplies can be in the millions if a large city has been hit. This presents a formidable logistics problem in which emergency response crews have to battle against the after effects of the disaster while they try and reach the people in need before they succumb to their lack of essential resources.

For this reason, research into the area of how to prepare for and respond to natural disasters is of universal importance and personal interest. The techniques developed through research of how to prepare and react to a natural disaster can be equally applied to a hurricane in the United States or an earthquake in Japan, just by updating the country specific infrastructure and parameter data set. Of these two areas: preparation and response, this article focuses on the response aspect. Preparation includes studying decisions like where to place supply points, how much supply inventory to hold, and other choices that are made pre-disaster. On the other hand, by examining the area of response techniques, we will be looking at how to actually distribute the supply from the facilities to the areas that need it.

Information about the roads and other infrastructure in these countries might be readily available, but forming a response plan strictly based off these numbers could lead to a solution in which the route given includes roads that have been damaged by the disaster and are now impassable. This could lead to large deviations from the original planned arrival times to the different cities that the route encompassed. This could mean people dying or disease spreading (if they have to drink from a contaminated water supply to survive) due to not receiving the needed relief supplies within time.

In this paper, we focus on formulating a response strategy that takes into account the uncertainties that arise in disaster situations. As stated earlier, one example of this is the uncertainty that a road which pre-disaster could be traversed by a semi-truck, could still be traversed by the same truck or if it could only support a smaller vehicle or no vehicle at all. In an area where the ground underneath the road could

shift during a flood leaving only one lane still intact would be a real life example of this change in road size capacity. In addition, we will also look at the uncertainty of travel time between locations in order to represent the chances that a road with a known travel time, could now take longer to travel due to debris littering the pavement. Uncertainty is not just limited to road conditions as in a disaster; the exact amount of supplies needed in and the time it will take to serve an area can be grossly under/overestimated during the chaos that ensues. For example, if you bring only enough water for the reported number of people who are unable to access clean water what will happen if the reported number is wrong? If it is too low, serious health consequences for the people could follow. On the other hand if it is too high, the water could sit in the vehicle and miss being distributed to someone else in a location that desperately needed it.

One of the main aims of this research is to develop models and algorithms for planning the delivery of relief supplies that take into account the information uncertainty that could accompany a natural disaster situation. In the chaos caused by a disaster situation, determining precisely if some of the roads have been affected by the natural disaster and to what extent can be a formidable task that responders may not have the leisure to pursue. However, information about what the usual road conditions are (optimistic estimate), a worst case (pessimistic) estimate of what could happen, and probability of each instance would probably be readily available. Using this information an expected value could be obtained and used in the planning process. Although we recognize only the most conservative plan would be able to handle a worst-case scenario and that using the expected values does not guarantee that the plan will be able to this worst-case scenario, we believe it is a good way to balance the tradeoffs associated with purely optimistic and purely pessimistic plans in a disaster situation.

By introducing uncertainty and reported information into the problem, the issue of how to determine reliability of this reported information arises. If we get information about the condition of a road from a Twitter update versus a news report how reliable is each? If the news report is twice as reliable as the Twitter update, does that make the reliability of the news 80% and the tweet 40% or is the news 90% and the tweet 45% reliable? The discussion on deciding how reliable the information is could be another research topic completely on its own. For simplicity and to keep the focus on the response aspect of emergency management, we have assumed that the reliability of the reports are known and given to us so that we can focus on the creation of vehicle routes.

In our model the supply points (depots) from which the vehicles leave and return have already been decided and we will focus on the route creation aspect of the emergency response process. As previously stated, we have decided to use the expected value for each of the uncertain quantities when running the model in order to hedge our bets against the optimistic and pessimistic scenarios. If we used just the optimistic values to plan a route then there could be drastic underestimates, thus resulting in negative consequences for the people awaiting service. On the other hand if we assume everything is going to be absolute worst-case, then the plan might have unnecessary precautions that end up costing the responders precious time or other things. Using the expected value is intended to balance out these risks so that the prescribed plan is still a good solution in light of the uncertainties that might occur when it is carried out. Some might argue that the expected value is a bad value to use as your uncertain travel time for example usually could never equal the expected travel time for that road, but we believe that it will give the model a better representation of what could be experienced on a route than using either of the end values (optimistic and pessimistic case) . If you start making assumptions about what the travel time will be by just using the optimistic or pessimistic value, i.e. Road-A will be fine and Road-B will be blocked off, then your solution is only optimal if Mother Nature makes those assumptions a reality in her path of destruction.

Below is an example of why we want to include uncertainty in our model and how its inclusion can lead to different results than what a normal delivery tour would suggest. In Figure 1-a, the model is not taking into account uncertainty and the travel time (in hours) that it takes you to traverse a road is unaffected by the uncertainty in road conditions that a natural disaster can cause. If using the popular VRP objective of minimizing total travel time, the optimal solution is bolded. Figure 1-b shows how this solution would change with the introduction of uncertainty. The top road could be near a river that has a high chance of being flooded which would take a truck much longer to travel on, so this is reflected in the chance of the

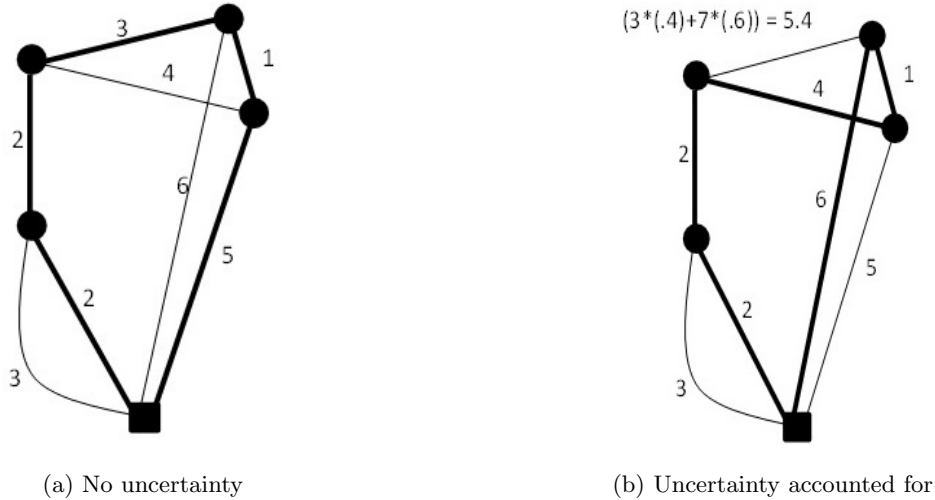(a) No uncertainty   (b) Uncertainty accounted for

Figure 1

increased travel time with probability 0.4 it can still be traversed in 3 hours, but with probability 0.6 it will take 7 hours. By using the expected travel time which is , the optimal solution for a minimizing total travel time problem would now call for a cross over, which is quite different to what it would have been without the element of uncertainty. This result is due to the triangle inequality, which states that the shortest distance between two points is a straight line, is no longer valid in the scenario with uncertainty so you can potentially arrive earlier by not taking the direct route.

In the following report we will first give a review of the relevant literature pertaining to the areas of disaster relief and planning for supply routes with uncertainty. Then we will introduce parts of the model that are universal to all of the problem types before exploring the specific problem types, and how the different areas in which uncertainty exists could affect how the optimal solution. Finally, we will discuss algorithms that we believe are well suited for use on this type of problem and briefly touch on a route simulation that we carried out and what its results tell us.

2   Literature Review

Since the problem that we aim to tackle falls under the vehicle routing category, The Vehicle Routing Problem by Toth and Vigo (2002) provided a good knowledge base for which to learn what exactly this type of problem included. Chapter 1 in particular defined the different types of vehicle routing problems (VRPs) that could be experienced in the real world. The different characterizations of these models were reflected upon the while the problem in which this paper examines was being created. Chapter 2, which defines mathematically some basic VRPs, was useful in providing a starting point for which to create the specific models for this research while also providing a base to compare mathematical logic against.

In the specific context of disaster management and logistics, many papers were available due to the popular and universal nature of disaster relief. A large number of researchers focused on the issue of facility location. Huang et al. (2010) addressed the p-center problem in a situation where a whole city would be functionless. In this case, the facility at the city would be unable to respond to its own emergency so they sought to use a dynamic programming approach to find the optimal locations for facilities on a general network. Duran et al. (2011) examines a similar problem in the optimal location of resource stockpiles worldwide in order to reduce the emergency response time. Still in the realm of facility location, Van Wyk et al. (2011) studied the best location for supplies so that inventory costs are minimized. This article also recognizes the uncertainty that comes with disaster and used a stochastic model to address this. Advar and Mert (2010) also see the connection between disasters and uncertainty and formulate a model that seeks to

minimize the total costs while maximizing the credibility of the international agencies that help in worldwide disaster planning/relief.

In regard to vehicle routing problems, Suzuki (2012) discusses the problem of having to respond to a natural disaster with a limited amount of gasoline to support the fleet. The analysis tries to determine if a shortage of fuel damages the logistical goals more than an equivalent-sized shortage in supplies, and when facing a limited fuel supply what type of vehicles should be used. Balcik et al. (2008) look at the routes created by vehicles leaving pre-determined distribution centers when trying to maximize benefits to aid recipients and minimize transport costs. Nolz et al. (2011) examine a problem very similar to the one we have defined in this paper. In their problem, Nolz et al. acknowledge the risks associated with assuming the infrastructure is the same post-disaster and use this in trying to identify solutions that would allow for the regular delivery of certain amounts of clean water to locations where the surrounding population can come and access the water. In order to solve this problem, Nolz et al. (2011) use multiple objectives with the first objective measuring the amount of risk, defined as the possibility that delivery tours become impassable after natural disasters. The second objective measures the amount of coverage, number of people who they believe can come to the water drop points, which the logistical system of routes and drop off points contains. The final objective simply measures the total travel time that it takes to complete the routes between the water drop point and depot. They then run different scenarios and compare the tradeoffs between the objectives. For example, one solution might have a higher risk, but have a greater coverage where another solution might sacrifice coverage for a lower travel time. These papers offer valuable insight to the types of problems that are faced in emergency management planning, place the problem which we are analyzing in context to the larger picture. The papers on VRPs are particularly helpful in highlighting issues that others have faced in their research that we might encounter in ours, and by offering solutions to these issues.

3 Modeling

There are many challenges associated with this area of research. First, it is often unclear what the goal or objective of the model should be. In a business environment, minimizing total travel time is usually a good goal because drivers are paid by the hour and the cost of paying for that labor can be minimized with a travel time related goal. Vehicles are also burning fuel every minute they are away from the depot so the longer they are away from the depot driving, the more fuel costs a company incurs. But when you switch away from the making profit mindset and instead are focusing on saving lives / responding to a disaster what should the goal be? Minimizing total travel time might yield a good solution but in a disaster situation is the time that a vehicle is away from the depot the key value to judge the solution on or would an objective that focuses on the amount of time it takes response crews to arrive at the last location in need a better goal? If the latter seems to align more with the mission of response crews that are trying to distribute supplies to reduce the number of casualties then maybe a goal that minimizes the time of arrival at the last location in the tour would be the best decision. Another goal that might be better for a disaster model than a business model is serving as many locations as quickly as possible. In principle this is a good goal, but defining how it will be measured is quite difficult. If you want to serve as many locations as you can within the first eight hours you may get a solution that serves four locations within this time, but then the fifth location might not get served until another eight hours later due to the poor position of the vehicle at the end of the initial eight hours. Figure 2 illustrates this through comparing a route that serves four locations as quickly as possible with a route that serves three locations as quickly as possible. The example on the left serves four nodes as quickly as possible and does so by traveling to the left nodes first, thus reaching the fourth node when time equals 7. In contrast, the time it would take the vehicle to reach four nodes if it traveled to the nodes on the right hand side first is 8. However, if the objective becomes to reach three nodes as quickly as possible then traveling to the right side first becomes optimal and three nodes can be reached by time 5. Traveling to the left first is no longer optimal as it would take until time 6 for the vehicle to reach the third node on the left.

Serve 4 Locations as Quickly as Possible   Serve 3 Locations as Quickly as Possible
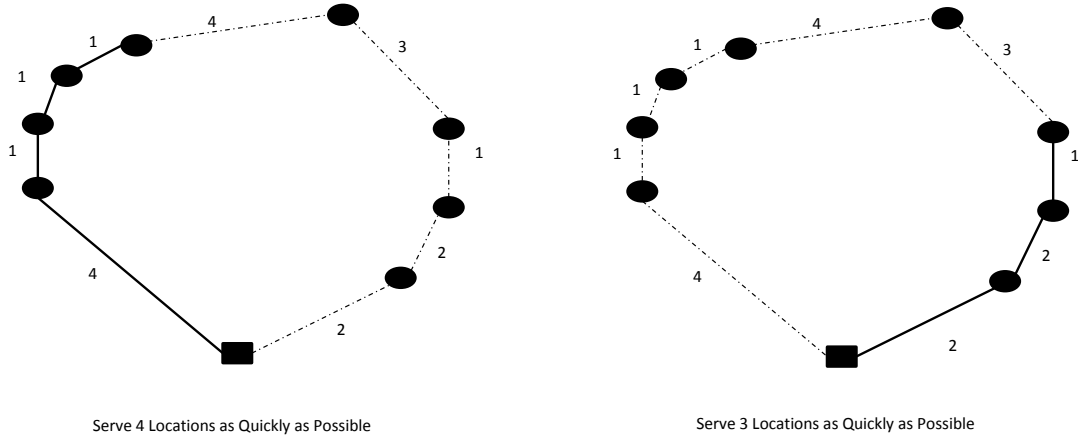
Figure 2

If you make the time period too long in a serve as many as quickly problem, greater than or equal to forty-two in the previous example, and all locations can be served within the time limit, then the problem would become a minimize the arrival time to the last location problem as discussed earlier. Additionally, is it better to serve as many locations as quickly as possible or serve as much demand as quickly as possible? If you just look at as many locations as possible you may visit four small towns with only 100 people in need in each city first, then arrive much later at a large metropolitan area with 10,000 people in need later which may result in a higher number of casualties that if you visited the 10,000 people first. As you can see, defining what the goal for a disaster response should be is a difficult and may depend on the situation on the ground.

We model the problems on a directed graph $D = (V, A)$ where V is the set of nodes that represent the different locations that need to be visited, and where A represents the arcs or roads between the locations and the arcs within A are defined by (i, j) for an arc that leaves location i and goes to j. The vehicles available to use are included in the set of vehicles which is given by K.

In the models there are various parameters such as the maximum amount or capacity that vehicle k possesses which is represented by C_k where $k \in K$. All of the vehicles also leave from and return to one central depot which is to be defined in the model as node $i = 0 = n+1$ where $|V| = n$, 0 is the depot when a vehicle leaves it, and $n+1$ is the notation for the depot upon a vehicles return. In all cases there are travel times associated with each arc. In cases where the travel time is known for certain the travel time from node i to j along arc (i, j) is given by t_{ij}. When there is uncertainty in the travel time, t_{ij} represents the optimistic travel time where the pessimistic travel time is given by $\bar{t_{ij}}$. The same logic applies to the demand of each node. When there is no uncertainty d_i represents the demand at node i, but when uncertainty of demand is introduced it represents the optimistic case and $\bar{d_i}$ is the pessimistic value. Each node has a value for the time it takes to complete serving a node (ex. dropping off supplies) and this is given by s_i. Like travel time and demand when uncertainty is introduced s_i represents the optimistic case where $\bar{s_i}$ gives the pessimistic service time value.

We model uncertainty by using two values (referred to as optimistic case and pessimistic case) for the parameter that is unknown, and by associating a probability with each of the values. The probability that the optimistic value is correct over the pessimistic value is given by p_* where * is the matching subscript to the parameter in question. For example if travel time is uncertain, then the probability that the travel time of arc (i, j) is t_{ij} is denoted by p_{ij}. This implies that the probability of the travel time arc (i, j) being $\bar{t_{ij}}$ is denoted by $(1 - p_{ij})$, and by using these definitions the expected travel time of arc (i, j) is given by $p_{ij}t_{ij} + (1 - p_{ij})\bar{t_{ij}}$.

5

In order to keep track of the routes that the vehicles follow and to check which nodes have already been visited we need a variable that shows whether a vehicle has been to a node. The way we have decided to do this is through the use of a binary variable x_{ijk} $i, j \in V \cup \{0\}$ $k \in K$ that takes on the value 1 if vehicle k travels from i to j. As many objective functions are time based (minimum travel time, etc) there needs to be a variable that keeps track of the time at which a vehicle arrives at each node before it begins service. In our model this variable is represented by z_{ik} $i \in V$ $k \in K$ where z_{ik} is the time at which vehicle k arrives at node i.

In the models, we want exactly one vehicle to visit each node therefore we need constraints that control the routes. Constraint (3.0) sums x_{ijk} across all vehicles and across all the arcs that leave i (given by $(i, j) \in A : j \in \delta^+(i)$) and that has to equal one for each node i. By setting it equal to 1, exactly one vehicle will visit i.

Constraint (3.1) defines the route by saying that the same vehicle that enters a node must leave the node. The first sum sums x_{jik} across all the arcs that enter i so that x_{jik} takes on the value of 1 when vehicle k comes from some node j to node i. This summation, which will be equal 1 because of constraint (3.0), has the summation of the arcs that leave from node i subtracted from it and is set equal to zero. If in any case x_{jik} equals 1, by vehicle k traveling from j to i, then x_{ijk} must also become 1 as bound by the constraint.

(3.0) $\quad \sum_{k \in K} \left(\sum_{(i,j) \in A : j \in \delta^+(i)} x_{ijk} \right) = 1 \ \forall \ i \in V$

(3.1) $\quad \sum_{(j,i) \in A : j \in \delta^-(i)} x_{jik} - \sum_{(j,i) \in A : j \in \delta^+(i)} x_{ijk} = 0 \ \forall \ k \in K, \ \forall \ i \in V$

Vehicle capacity cannot be exceeded therefore constraint (3.2) is needed. The summation of the values of x_{ijk} is driven by $(i, j) \in A : j \in \delta^+(i)$ and follows the same logic from the previous two constraints. Due to the multiplication between $\sum_{i \in V} x_{ijk}$ and d_i the demand, the left side becomes a running total of all the demands at for the nodes that k visits as the product of the two only takes on a non-zero value when $\sum_{i \in V} x_{ijk}$ equals one, meaning that vehicle k visits node i. This value has to be less than the total capacity available for the vehicle or the vehicle would run out of relief supplies in our disaster example so the right side is set to C_k.

(3.2) $\quad \sum_{i \in V} d_i \sum_{(i,j) \in A : j \in \delta^+(i)} x_{ijk} \leq C_k \ \forall \ k \in K$

The values that variable x_{ijk} can take on must be specified, therefore constraint (3.3) makes x_{ijk} a binary variable.

(3.3) $\quad x_{ijk} \in \{0, 1\} \ \forall \ i, j, \in V, \ \forall \ k \in K$

In addition to constraints that control the structure of the routes, constraints are needed to create and calculate the time aspects of the model as most popular objectives for supply distribution are time related. Constraint (3.4) makes our time variable, z_{ik}, continuous and greater than zero.

(3.4) $\quad z_{ik} \geq 0 \ \forall \ i \in V, \ \forall \ k \in K$

(3.5) $\quad z_{ik} + t_{ij} + s_i - z_{jk} \leq M(1 - x_{ijk}) \ \forall \ k \in K, \ \forall \ (i, j) \in A$

In the constraint above (3.5) which is used as a relationship constraint to make z_{ik} and z_{jk} take on the correct values, the letter M is the sum of the $n+1$ largest travel times plus the n largest service times where n is the number of nodes to be visited. M is set as this number because the constraints in the problem make it where each arc/node can only be traveled/visited once, therefore the number of arcs used to visit n nodes and return to depot will be $n + 1$. Since only $n + 1$ arcs will be used, and each arc used only once, it is

impossible that the total travel time of these arcs can exceed the sum of the $n + 1$ largest arcs. Service time of the previous node is also accounted for in the constraint so in order to ensure z_{ik} does not take on a value larger than M is not possible the n largest service times are added as well. This makes sure that no matter what service and travel times a vehicle experiences, it will not be able to take on a value greater than M. The value of M is multiplied by $(1 - x_{ijk})$ because if a vehicle k travels from i to j then x_{ijk} will take on the value of 1 and the right side of the constraint will become 0. This forces the time when a vehicle arrives at node j, or z_{jk} to be greater than or equal to the time the vehicle arrived at node i plus the service time of node i and the time it took to travel from i to j for the constraint to be satisfied. If vehicle k does not travel from i to j, then x_{ijk} will equal 0 and the right side of the constraint will be equal to M. The constraint will then always be satisfied no matter what values that z_{ik}, the service time, the travel time, or z_{jk} take on as sum of the first three will not be required to equal z_{jk} when vehicle k does not travel from node i to j. This constraint stays structurally the same throughout the different models, and only the values for the service and travel times might be changed to an expected value based off whether they are the uncertain parameter in question.

In all cases we are going to examine, the objective of minimizing the time of arrival at the last location is seen as superior to the objective of minimizing total travel time. This is because, as previously stated, in a disaster scenario it would be more relevant to try and minimizing the time it takes to reach all locations in need than it would be to just minimizing total travel time. This means that the popular minimize total travel time objective function, $\sum_{k \in K} (z_{n+1,k} - z_{0,k})$ needs to be altered to represent minimizing the time of arrival at the last node. To correctly represent this goal, the objective function needs to be changed to $\sum_{k \in K} (z_{n+1,k} - \sum_{i \in V} (t_{i,n+1} + s_i) x_{i,n+1,k})$. Because of the definition of the depot at $i = 0 = n + 1$ the equation $z_{n+1,k} - z_{0,k}$ gives the total time away from depot for vehicle k, and because the objective of minimizing time of arrival at the last node will be seen as better than the minimize total travel time objective, vehicles will want to leave the depot at time 0 because waiting past then will just hurt the objective function value; therefore, the value of $z_{0,k}$ goes to 0 which is why it has been left out in the new objective function. To get the time of arrival the last node, you need to subtract the travel time from the last node back to the depot, and the service time of the last node from the total time away from the depot. This is given by $\sum_{i \in V} (t_{i,n+1} + s_i) x_{i,n+1,k}$ which only takes on a non-zero value (the sum of the travel time from node i back to the depot, and the service time at node i) if $x_{i,n+1,k}$ equals one. Because of the route constraints defined earlier that each node will be visited by only one vehicle, each node can only have one vehicle leave it, therefore $x_{i,n+1,k}$ can only take on the value of one for one node i for each vehicle k. To get the time for the entire fleet you then sum over K.

Now that the foundation for all problem types has been discussed, we will in the next sections examine the different problem variations and analyze how the optimal solutions may change when the objective function for the model is altered, and when uncertainty is introduced.

3.1   Dire Need Variant

In this problem type, the nodes are assumed to have different need urgencies therefore the best solution would serve a node with a higher need before it served a node with a lower need. The need of a node is represented by the parameter N_i for $i \in V$ where a lower value of N_i signifies a more urgent need. This type of problem could be easily applied to real world scenarios as all areas during a disaster are not affected the exact same. Using hurricane Katrina as an example, the cities closest to the gulf suffered greater damage than cities further inland as the strength of the hurricane died down after landfall. Although cities on the gulf and further inland both may need relief supplies, the gulf area might be in more urgent need due to the greater destruction in its area whereas the inland locations may need minimal relief. When deciding how to serve these areas, trying just to minimize a time oriented objective function may put the inland area first and therefore postponing service to the gulf location, which could lead to numerous casualties in the urgent care area. If the objective was need based, then these casualties could possibly be avoided and the area that needs relief the worst would get served first. When modeling this, the issue of how to calculate N_i arises.

Should it be on a scale from 1-10 with general descriptions for each value, or a large specific scale in which the values from 1-100 are given based on very specific circumstances for each value? For this project we will assume that the value of N_i for each node has already been calculated.

Our objective in the problem is to serve all of the nodes while trying to serve the nodes with the most need first. This objective can be illustrated by

$$\min \sum_{i \in V} \sum_{k \in K} \frac{z_{ik}}{N_i}$$

Because N_i is the divisor, the larger the value of z_{ik} becomes, the larger you would want N_i to be in order to produce a smaller overall value. It is this logic that should drive the objective function to pair high needs with lower (therefore earlier) service times.

Changing the objective function from minimizing travel time to one that includes need in the objective can cause changes in the optimal solution even when there is no uncertainty present. Figure 3 shows an example where changing the objective from minimizing the time of arrival at the last location to the one specified in this section can alter the optimal route. As seen in Tables 1 and 2, when you use the minimizing arrival time at last node, the sum of the arrival times of each node divided by the need of each node is greater than the route shown on the right by .5.
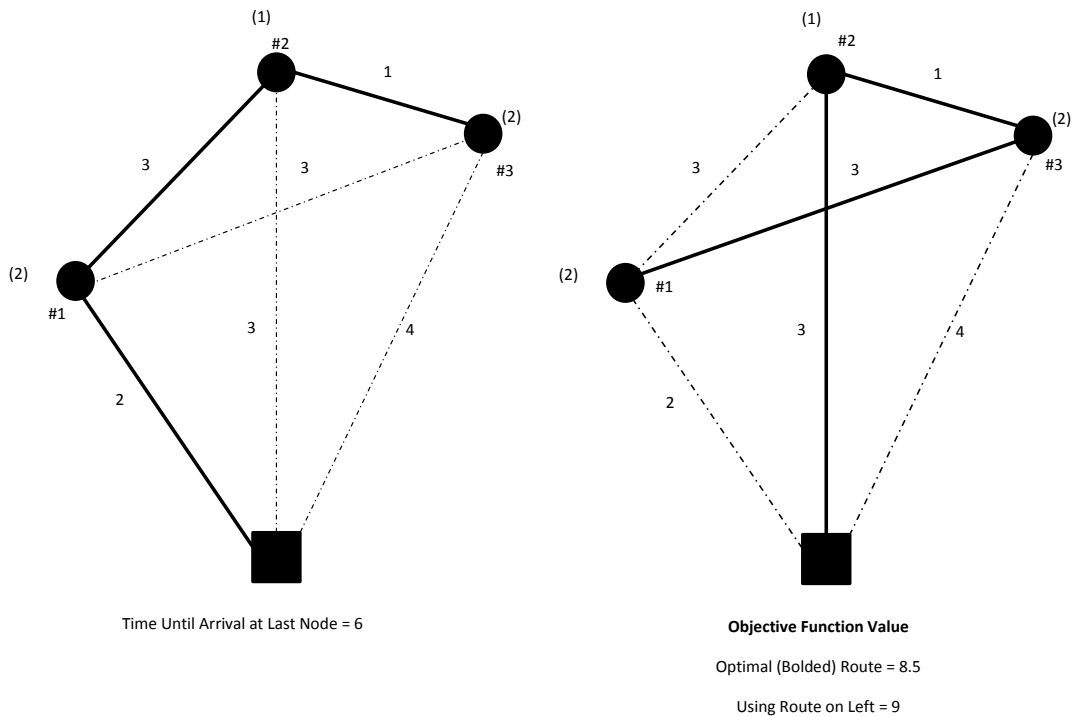

Figure 3

Dire Need Objective Using Travel Time Route			
Node Number	#1	#2	#3	
Arrival Time at Node	2	5	6	
Need of Node	2	1	2	Objective Value
z_{ik}/N_i	1	5	3	9

Table 1

Dire Need Objective Optimal Route			
Node Number	#1	#2	#3	
Arrival Time at Node	7	3	4	
Need of Node	2	1	2	Objective Value
z_{ik}/N_i	3.5	3	2	8.5

Table 2

One of the most obvious areas in which uncertainty could have an effect on the optimal solution in the dire need model type is uncertainty regarding the value of N_i given for each node. In Figure 4 uncertainty is introduced for the value of N_i, and for each location the value of N_i is split 50%-50% between the two values in parentheses. In the left example in Figure 4, the value for each node's need is the same as it was in the previous example without uncertainty. The example on the right of Figure 4 shows the optimal route if all the locations in the example could take on the value of 1, with the location at the top having a significantly higher optimistic case value (pessimistic being the lower, therefore direr need case). As you can see in the bolded optimal solution, the model suggests that it is better to serve the locations with the lower (more urgent) optimistic case first and serve the top node last it could potentially have the least urgent need. Tables 3 and 4 break down the arrival time and expected need of each node, and give the respective objective function values for each route. In this example, the route suggested is different than any of the routes previously suggested. This shows the variation in the optimal solution that can occur when the need of the location is the uncertain parameter.
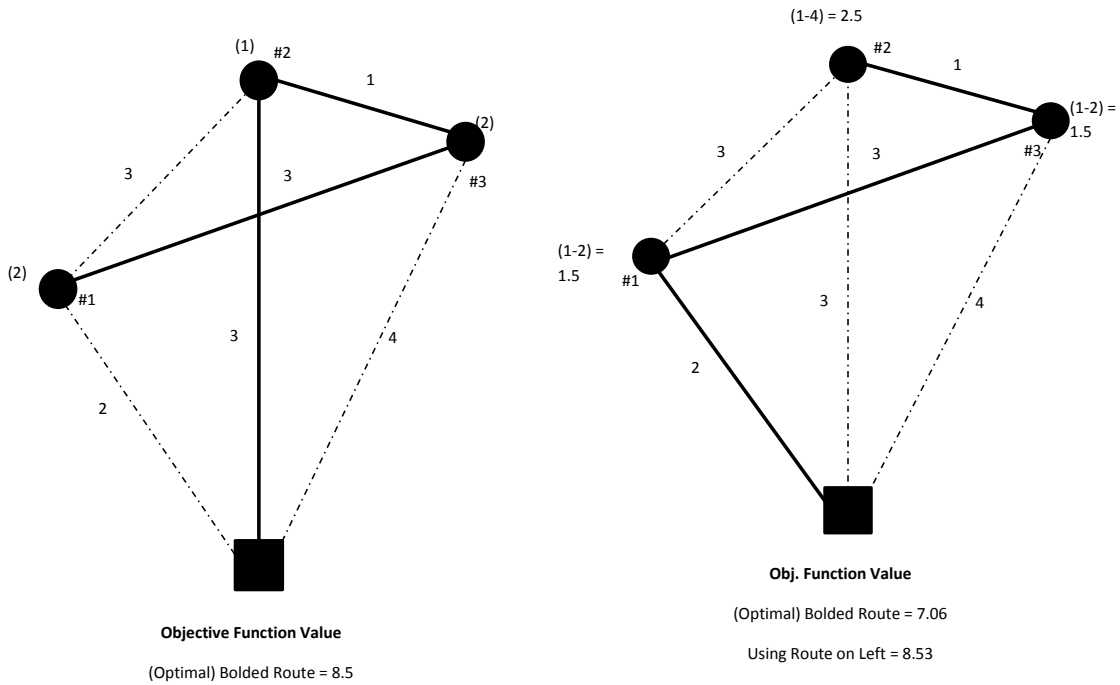

Figure 4

Dire Need Objective No Uncertainty Optimal Route			
Node Number	#1	#2	#3	
Arrival Time at Node	7	3	4	
Expected Need of Node	1.5	2.5	1.5	Objective Value
z_{ik}/N_i	4.66	1.2	2.66	8.53

Table 3

Dire Need Objective With Uncertainty Optimal Route			
Node Number	#1	#2	#3	
Arrival Time at Node	2	6	5	
Expected Need of Node	1.5	2.5	1.5	Objective Value
z_{ik}/N_i	1.33	2.4	3.33	7.06

Table 4

3.2 Serve As Many As Quickly As Possible

In this problem type, all nodes have the same urgency of need and the goal is to serve the most nodes as quickly as possible. In the solution it would be better if you could serve 5 nodes in 5 hours with a 7 hour total travel time than if you could serve 5 nodes in 6 hours with a 6 and a half total travel time.

As previously discussed, defining what "serve as many as quickly as possible" means is a difficult task. In this variation we explore an alternative "serve as many as quickly as possible" objective which focuses on serving as many locations as possible in a given time period. Figure 6 illustrates how uncertainty could affect the optimal route suggested by the serve as many as quickly as possible objective. In both examples the goal is to serve as many as possible in the first six hours. In the left example there is no uncertainty and if the vehicle visits the locations in the order of 1-2-3-4, then all of the nodes can be reached within the first six hours. If they are visited in any other order, then there is no way to reach all nodes within the first six hours. In the right example travel time uncertainty is introduced and by using the expected values, a new optimal route is created. In order to visit the most nodes possible within six hours, the optimal route is 4-3-2. If you were to try and use the optimal route order from the example on the left then in the first six hours one could only expect to reach 2 nodes. It is also important to note that if the 4-3-2 route is taken and both of the first two arcs take on their worst case value (4 and 2), then it is still possible to visit two nodes, whereas if one travels the route 1-2 and both of the first two arcs in that route take on their worst case value (4 for both), then you would only reach one node in the first six hours.
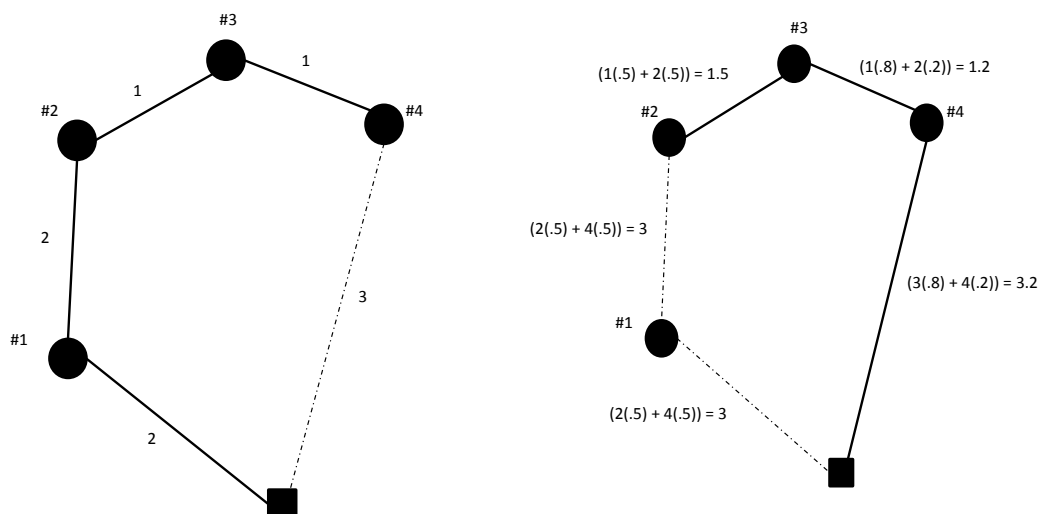

Figure 6

In order to count the number of locations that are visited before the specified time, which will be defined as H, we need to introduce the binary variable y_i for all $i \in V$ which takes on the value of 1 if node i is

visited after time H and 0 otherwise. The objective function shown below maximizes $(1 - y_i)$, so the variable will want to become 0 in as many instances as possible. For y_i to be able to take on the value of 0, z_{ik} must be less than H, which in the problem means that the vehicle k will have to arrive at node i before time H. This is modeled in constraint (3.2.0). If z_{ik} is less than H there is no reason to add M to it, and y_i will take on the value of 0 (M is the sum of the $n + 1$ travel times plus the n largest service times and is used so that when added z_{ik} cannot be greater than $H + My_i$). However, if z_{ik} is greater than H, then for the constraint to hold true y_i must take on the value of 1 so that M can be added to H. Therefore, the route which visits the most nodes before time H will allow for y_i to take on 0 in the most instances and be the optimal route.

$$\max \sum_{i \in V} (1 - y_i)$$

$$(3.2.0) \quad z_{ik} \leq H + My_i \ \forall \ i \in \ V, \ \forall \ k \in K$$

The value of z_{ik} depends on the route taken and must be modeled in constraint (3.2.1) where $(p_{ij}t_{ij} + (1 - p_{ij})\bar{t}_{ij})$ gives the expected travel time from i to j along arc (i, j)

$$(3.2.1) \quad z_{ik} + s_i + (p_{ij}t_{ij} \ + (1 - p_{ij})\bar{t}_{ij}) - z_{jk} \leq M(1 - x_{ijk}) \ \forall \ k \in K, \ \forall \ (i, j) \in A$$

3.3   Service Time Uncertainty

In this problem type, the time that it will take a vehicle to serve a node, drop off humanitarian aid, is not known with 100% certainty. For example, in the real world when a vehicle goes to deliver supplies to a large city, it may have to make more than one stop in that city as it visits multiple drop off points within the city. The time it takes to visit these distribution sites within the city and deliver the supplies would be the service time. If the disaster has affected transportation infrastructure in a significant way, this tour of the supply drop off points may take much longer than predicted thus increasing the service time. In this variation we seek to analyze how an uncertain service time affects the optimal route, or if it even affects it at all.

Service time is an important aspect to take into account, because even when it is not affected by uncertainty it can cause significant changes to the route when not accounted for. In Figure 7, the example on the left shows the optimal route when all the service times are assumed to be equal and take on a value of 0. However, when one of the service times becomes significantly larger than the others, that node becomes the node last visited. This is shown in the example of the right of Figure 7 where node #1 has a service time that is greater than the other nodes by 3, thus making it the last node on the tour when minimizing arrival time at the last node is the objective (the objective function values are shown below the diagram). Because it can have such impacts on the route, it is important to examine uncertainty in service time.

Objective Value for Bolded Route = 8

Objective Value for Bolded Route = 13

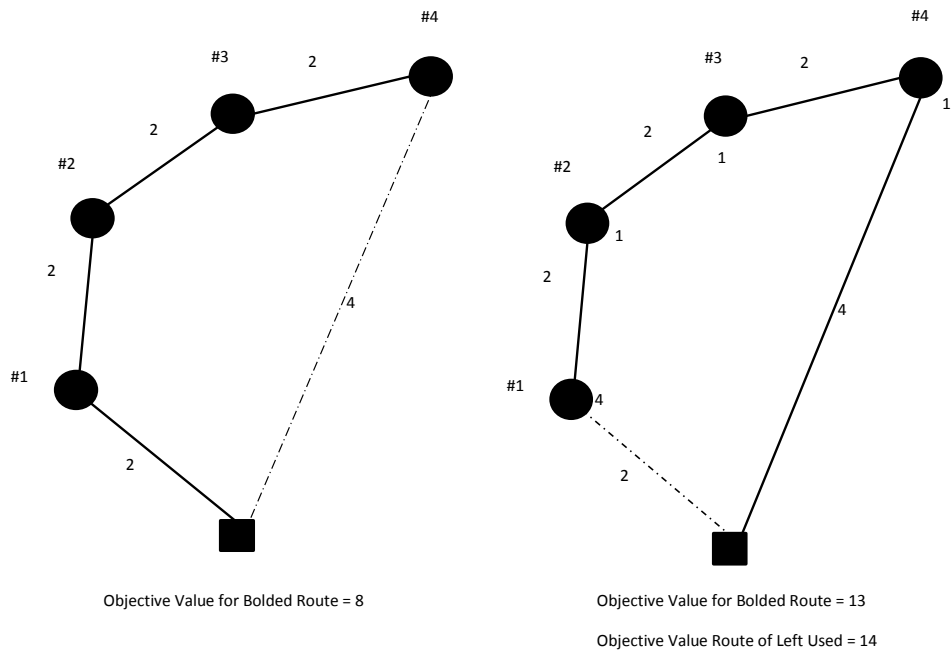Objective Value Route of Left Used = 14

Figure 7

In this variation, the service time is the only parameter that is not known with 100% certainty. The optimistic estimate for service time of node i is given by s_i and the pessimistic estimate is given by $\bar{s}_i$. The optimistic case is favored over the pessimistic case with a probability of p_i. While this type of uncertainty can be analyzed with many different objectives, the current objective in this example is to minimize the total time that it takes the vehicles to arrive at the last node on their route. Figures 8 and 9 below illustrate how uncertainty in the service time of a node can affect the optimal solution. When there is no uncertainty present (each nodes service time is known to be 5), the optimal tour travels in a counter-clockwise fashion stopping at every node in order as displayed in Figure 8. If uncertainty occurs, and the service time at the top node has a fifty-fifty chance of being 5 (optimistic) or 15 (pessimistic) then the route on the right becomes optimal as the expected service time is used as the service time for the top node as shown in Figure 9. Because there is a chance that the service time at the top node could take on a value that would greatly delay the time at which the other nodes get reached, it becomes optimal to save that node for last as to not risk delaying the vehicles arrival time at later nodes.
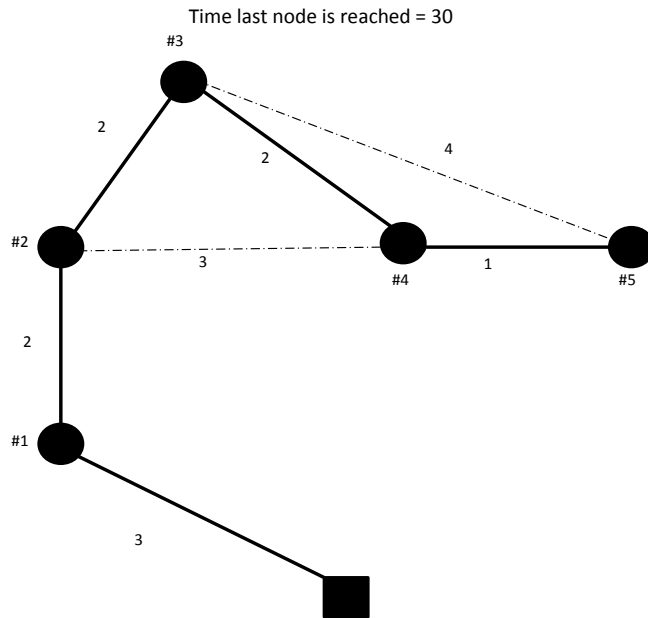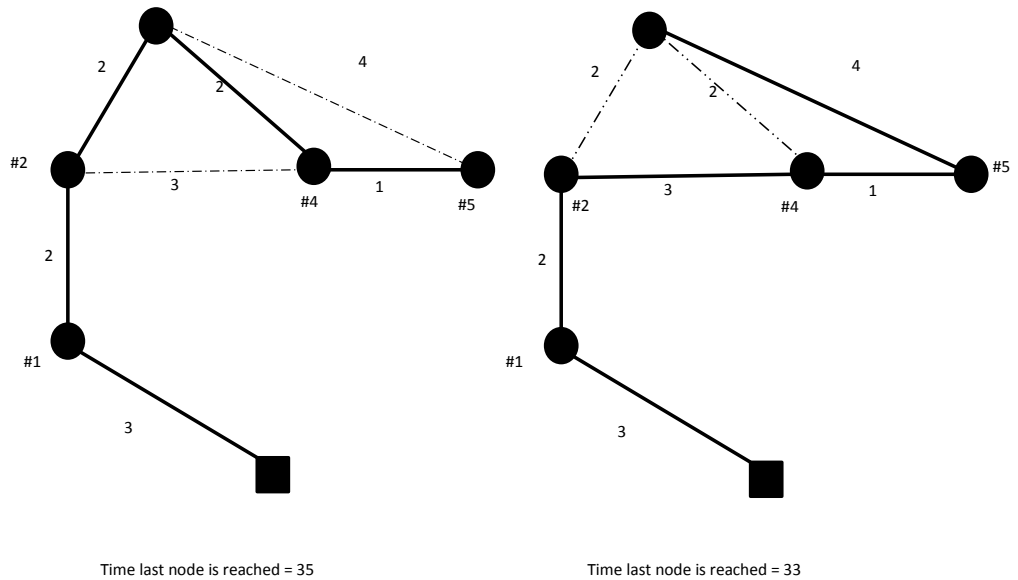
Figure 8

Figure 9

The objective is to minimize the total time that it takes the vehicles to arrive at the last node on their route while serving all nodes.

$$\min \sum_{k \in K} \sum_{i \in V} (z_{n+1,k} - (t_{i,n+1} + s_i) x_{i,n+1,k})$$

13

The value of z_{ik} depends on the route taken and must be modeled in constraint (3.3.0) where $(p_i s_i + (1-p_i)\bar{s}_i)$ gives the expected service time at node i.

$$(3.3.0) \quad z_{ik} + t_{ij} + (p_i s_i + (1-p_i)\bar{s}_i) - z_{jk} \leq M(1-x_{ijk}) \; \forall \; k \in K, \; \forall \; (i,j) \in A$$

3.4 Road Capacity Variant

In this problem type, the roads that connect locations are only able to be traversed by certain size vehicles. This is very applicable to the real world as not all roads in a disaster situation will be affected the same. Highways that are built far away from trees and do not have any bridges may have a lower chance of becoming unusable than smaller country roads. The country roads may have trees all around and a natural disaster may push large amounts of debris onto the road, and roads that utilize bridges may become significantly flooded so that both would become highly inaccessible. In this variation we want to examine how an optimal solution may change when the capacity of a road (or size of a vehicle that can traverse the road) is taken into account and not known with certainty.

Like in the other examples, uncertainty in road capacity or size of a vehicle that can transverse a road can be examined across a wide range of objective functions. It differs from the other problems in that in order to analyze it, we need to add some parameters that were not previously defined or discussed. To start, the most obvious parameter missing is road capacity itself. Road capacity along arc (i,j) when there is no uncertainty and for the optimistic case amidst uncertainty is shown by w_{ij} where $i,j \in V$. Using the same logic that the other parameters from early used, the pessimistic road capacity is given by $\bar{w}_{ij}$ where $i,j \in V$ and the probability the optimistic case in the actual road capacity going to be experience is given by p_{ij} and the probity of the pessimistic case is $1-p_{ij}$ respectively. For these parameters to play a role when forming the optimal solution, each vehicle needs to be given a size q_k and constraint (7.1) needs to be added. A vehicle cannot go down an arc that can't support its size so we must have the constraint (7.1). If a vehicles size - q_k - is larger than the allowable size for the arc - $(p_{ij} w_{ij} + (1-p_{ij})\bar{w}_{ij})$ - then x_{ijk} cannot take on the value 1 and the vehicle does not go down arc (i,j). All other constraints and parameters from the earlier definition of parts common to all problems are still in use.

Figure 10 and 11 illustrate how uncertainty in road capacity can lead to a different optimal solution than would be recommended with no uncertainty. In Figure 10, there is no uncertainty accounted for and the optimal route to serve all the nodes while minimizing the time you arrive at the last node is shown. When uncertainty in the road is taken into account and expected road capacity is used to formulate routes in Figure 11 the optimal solution changes. Based off the expected values, arcs/roads that would not be able to be traveled on with our vehicles are excluded so that a route that has a high chance of being impassable is not chosen. In the case with uncertainty, the middle arc that was left out in the first scenario is included, and the time arriving at the last node is only increased by 1. In a disaster response strategy, this small increase in time would be worth trading for the increased reliability of the route.

Demand at each node = 5

Travel time of the arcs dashed below = 3

Travel time of other arcs = 2

Capacity   Vehicle Size
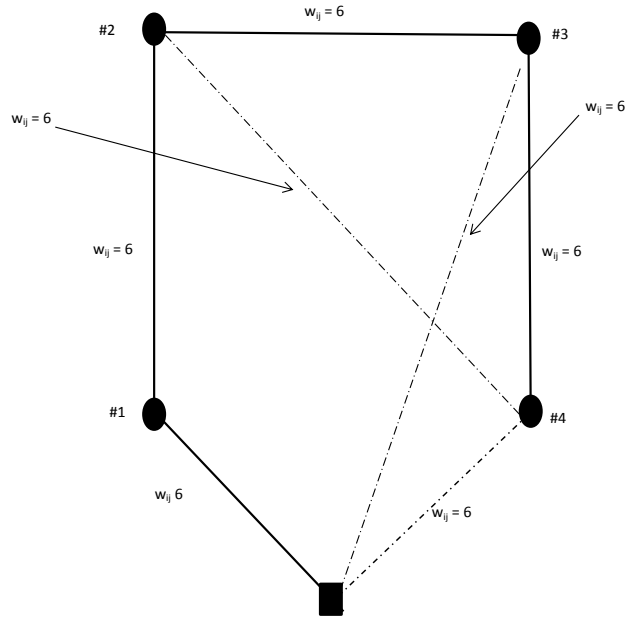
$C_1 = 20$ $q_1 = 5$

$w_{ij} = 6$

#2   #3

$w_{ij} = 6$ $w_{ij} = 6$

$w_{ij} = 6$ $w_{ij} = 6$

#1   #4

$w_{ij}\ 6$ $w_{ij} = 6$

Figure 10

$w_{ij} = (6(.5)) + 3(.5)) = 4.5$

#2   #3

$w_{ij} = (6(.7)) + 4(.3)) = 5.4$ $w_{ij} = (6(.9)) + 5(.1)) = 5.9$

$w_{ij} = (6(.9)) + 5(.1)) = 5.9$

$w_{ij} = (6(.8)) + 4(.2)) = 5.6$

#1   #4

$w_{ij} = (6(.9)) + 4(.1)) = 5.8$ $w_{ij} = (6(.7)) + 3(.3)) = 5.1$
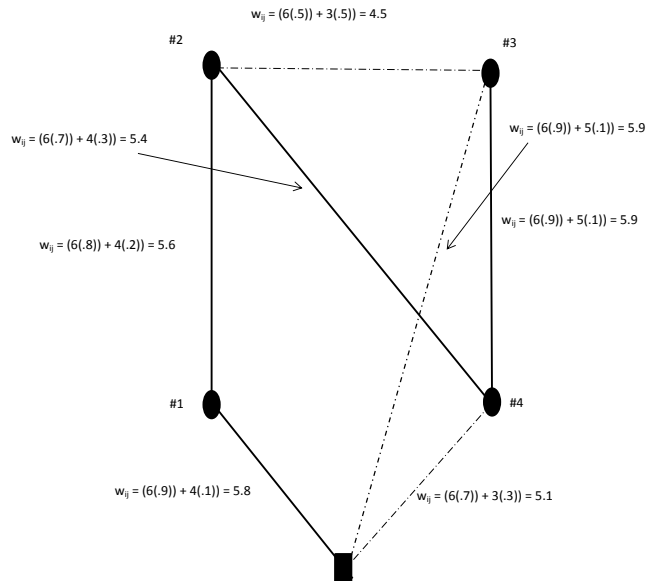
Figure 11

The objective, shown on the next page, is to minimize the total time that it takes the vehicles to arrive at the last node on their route while serving all nodes.

15

$$\min \sum_{k \in K} \sum_{i \in V} (z_{n+1,k} - (t_{i,n+1} + s_i)x_{i,n+1,k})$$

The value of z_{ik} depends on the route taken and must be modeled in constraint (3.4.0)

$$(3.4.0) \quad z_{ik} + t_{ij} + s_i - z_{jk} \leq M(1 - x_{ijk}) \ \forall \ k \in K, \ \forall \ (i,j) \in A$$

A vehicle cannot go down an arc that can't support its size so we must have the constraint (3.4.1). If a vehicles size - q_k - is larger than the allowable size for the arc - $(p_{ij}\bar{w}_{ij} + (1 - p_{ij})w_{ij})$ - then x_{ijk} cannot take on the value 1 and the vehicle does not go down arc (i,j)

$$(3.4.1) \quad q_k x_{ijk} \leq (p_{ij}w_{ij} + (1 - p_{ij})\bar{w}_{ij}) \ \forall \ k \in K, \ \forall \ (i,j) \in A$$

3.5 Uncertain Demand

In this problem type, the demand at each node is not known with 100% certainty. This is one of the easiest variations to imagine in the real world as the amount of supplies to take to each location in need is often not exactly known. Responders may know that a city needs water, but do they need 10,000 liters or 20,000 liters? It would be difficult to predict exactly how long it would be until the uncontaminated water utilities would be back online, and this could also affect the demand of the city as they wouldn't need a large amount of water from the emergency responders if clean water was going to be available a few hours after the responders left. Because of the natural uncertainty that comes with trying to estimate demand, in this variation we want to observe how this uncertainty could potentially affect the optimal route given by the model.

In a disaster situation the exact amount of humanitarian assistance needed is often not known, which makes studying uncertainty in demand a very applicable area. For demand uncertainty, the parameters for optimistic and pessimistic demand resemble the parameters for uncertain service time except that the letter s is replaced with a d. Figure 12 below shows the optimal route in a situation where the demand is known and exactly equal to the capacity of a vehicle ($C_k = 45$). In this case, it would be best to use one vehicle to cover all the demand if you wanted to minimize total travel time. However, if there was any chance that demand at one of these nodes would be greater than what is reported above then one would want to split this route into two routes between vehicles as any increase in demand would exceed vehicle capacity.
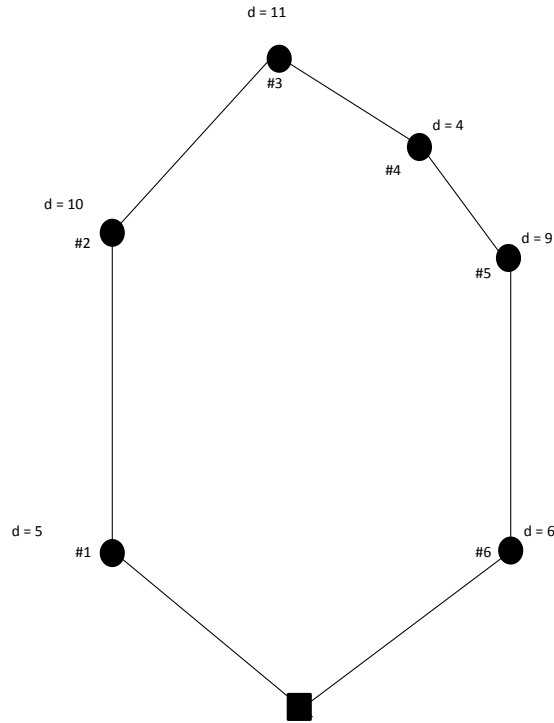
Figure 12

Figure 13 illustrates why studying uncertainty in demand can provide interesting results. In the two examples, the capacity of each vehicle is set at 20 and the travel time of the arcs not otherwise specified is 2. In the example with no uncertainty on the left, it is assumed that all the demands are known and the route that minimizes the time at which the last node is served is shown. In a disaster situation, an exact number for demand would probably be hard to attain. If you have reports of what it could be and use the uncertainty associated with this information you can calculate the estimated demand for each node and use this to build a route (which results in the route on the right). In comparison to the route suggested by the model on the left, when uncertainty is taken into account the time of arrival at the last node is increased by 3. This increase is considered necessary so that the entire expected demand can be met. Upon further analysis, by altering the routes in this way, both vehicles would still be able to meet all of their demand if half of the nodes they visit need the worst case scenario demand. If you use the routes suggested on the left figure and one of the furthest two right nodes has worst case demand, then demand at one of the nodes on the right tour will not be fully met. By using the expected demand, the routes created balance out the risks of running out of supply by only increasing the objective (stated earlier) by 3. The routes created in the right figure only increase the sum of the two routes (total travel time until all nodes are reached) by 1 from 14 to 15.
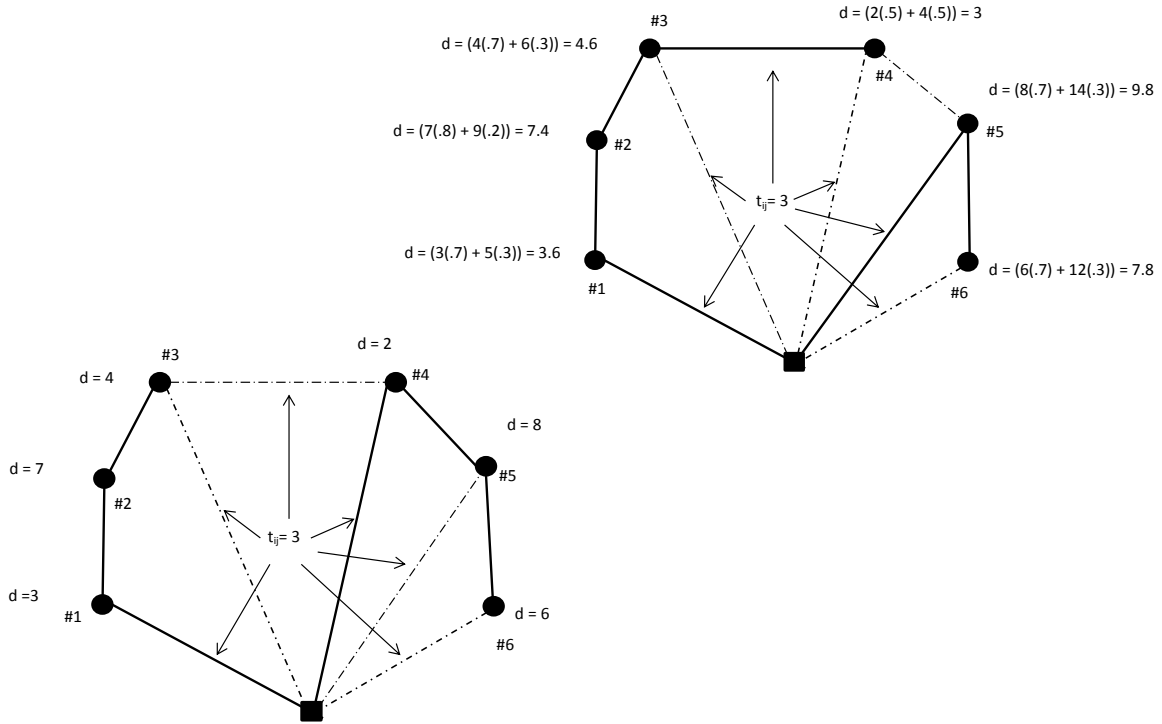
17

Figure 13

The objective is to minimize the total time that it takes the vehicles to arrive at the last node on their route while serving all nodes.

$$\min \sum_{k \in K} \sum_{i \in V} (z_{n+1,k} - (t_{i,n+1} + s_i) x_{i,n+1,k})$$

Because the demand is now uncertain, constraint (3.5.0) where the expected demand is given by $p_i d_i + (1 - p_i)\bar{d}_i$ will replace constraint (3.2) in this variation.

$$(3.5.0) \quad \sum_{i \in V} (p_i d_i + (1 - p_i)\bar{d}_i) \sum_{j \in \delta^+(i)} x_{ijk} \leq C_k \ \forall \ k \in K$$

The value of z_{ik} depends on the route taken and must be modeled in constraint (3.5.1)

$$(3.5.1) \quad z_{ik} + t_{ij} + s_i - z_{jk} \leq M(1 - x_{ijk}) \ \forall \ k \in K, \ \forall \ (i,j) \in A$$

4   Algorithms

The algorithm that could be used in creating routes for the different model types would be straightforward and incorporate a construction phase and an improvement phase. The construction phase would start from the depot, and if there were nodes that had not been visited then the node that would cause the smallest increase in the objective function would be added first. From that node, all nodes that had not been visited yet, had arcs that the vehicle was able to traverse, and whose added demand would not exceed vehicle capacity would be considered for insertion into the route. Of the nodes being considered, whichever node's insertion into the route created the smallest increase in the objective function value would be added. This process would continue until no more nodes could be added to the route (whether it be to vehicle capacity,

road capacity, or that all nodes had been served). If there were still unserved nodes, then go back to the first step and start at the depot with a new vehicle. If all nodes had been included into a route then the improvement phase would start. The improvement phase would start with multi-route improvement and string mix improvement would be used. This means that a string of one or two vertices would be moved from one route to another and that two strings of one to two vertices would be moved from one route to each other. If either of these techniques improved the overall solution then the change would become permanent. This would be ran numerous times until all of the options had been explored. Then an improvement within each route would occur when λ arcs are removed and then the λ remaining segments are reconnected in all possible ways to see if a profitable combination existed. This would occur for all routes and all arcs until all possibilities had been explored thus ending the improvement phase.

Incorporating uncertainty into this process would not be complicated as the uncertain values in the model could be easily calculated beforehand, and just substituted in for what would normally be known data. You would only need the optimistic and pessimistic value for the uncertain parameter and the probability of the optimistic case occurring (assuming that the chance of the pessimistic case would be one minus the chance of the optimistic case) so that it could calculate and use the expected value for the value of the uncertain parameter. This calculated expected value would be substituted in for the respective parameter in the place of data known with 100% certainty. Also, when analysing the cost to the objective function of adding a node, the time aspect of including the node would need to be the travel time of the arc to be traversed plus the service time of the node you are arriving at.

5 @Risk Simulation in Excel

In this research, models of networks were created in Excel in order to observe the effects of uncertainty on tours. Two networks of destination nodes and arcs were created, and travel times were assigned to the arcs for both networks. In order to study the effect of uncertainty in these networks through the use of scenarios where the travel time along an arc is uncertain, the arcs were also assigned a maximum, worst case, travel time and probabilities were associated with these increased travel times based on the likelihood of a vehicle that used that arc encountering the increased travel time. The illustrations of these two networks and the information for the arc numbers, travel times, probabilities, and expected value based off those probabilities are shown below in Figure 14 and Tables 5-6. (Note: The number next to an arc indicates the numeric label of that arc that is used in the tables.)
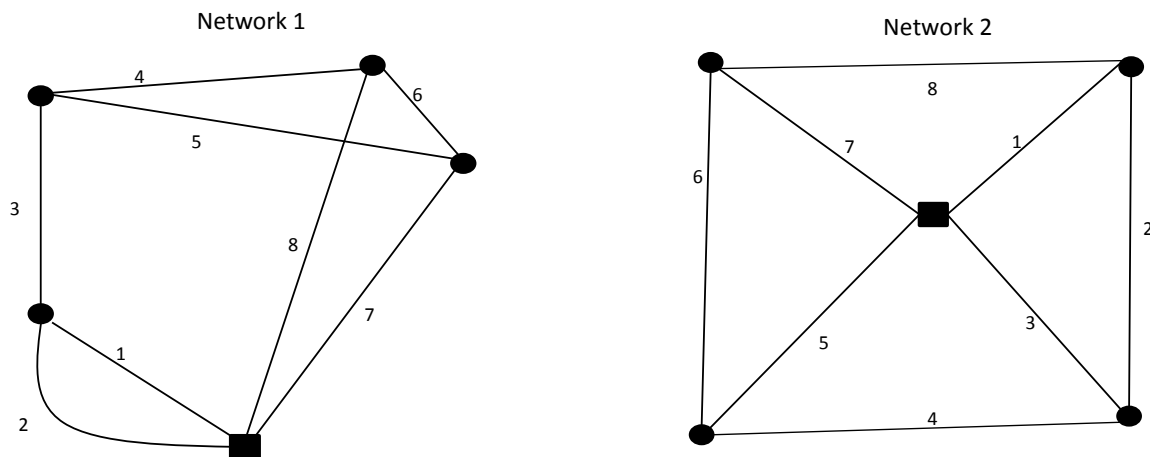

Figure 14

Network 1 Data					
Arc	Low	High	P(low)	P(High)	EV
1	2	5	0.4	0.6	3.8
2	3	4	0.5	0.5	3.5
3	2	3	0.9	0.1	2.1
4	3	7	0.4	0.6	5.4
5	5	6	0.8	0.2	5.2
6	1	2	0.9	0.1	1.1
7	4	7	0.2	0.8	6.4
8	6	7	0.8	0.2	6.2
Original Optimal Route: 1-3-4-6					
Travel Time w/o Uncertianty:8					
Optimal Route Using EV's: 2-3-5-6					
Travel Time w/o Uncertianty:11					

Table 5

Network 2 Data					
Arc	Low	High	P(low)	P(High)	EV
1	2	5	0.3	0.7	4.1
2	4	7	0.4	0.6	5.8
3	3	4	0.5	0.5	3.5
4	4	5	0.5	0.5	4.5
5	4	5	0.6	0.4	4.4
6	4	5	0.4	0.6	4.6
7	3	5	0.5	0.5	4
8	4	7	0.5	0.5	5.5
Original Optimal Route: 1-2-4-6					
Travel Time w/o Uncertianty:14					
Optimal Route Using EV's: 3-4-6-8					
Travel Time w/o Uncertianty:15					

Table 6

For each network, the average time of arrival at the last node in the tour and maximum time of arrival at the last node in the tour were recorded for two different routes. The first route used was the route that minimized the time of arrival at the last node in the tour when there was no uncertainty, and the second route used was the route that minimized the time of arrival at the last node in the tour when there was uncertainty and the expected value of the uncertain parameter was used. Both routes were initially analysed using a scenario in which the travel time was not known with 100% certainty and there was a discrete probability distribution between an optimistic and pessimistic value. This was accomplished by using the @Risk Simulation software tool which allowed for the parameters of the uncertain travel time on each arc to be entered, and then @Risk generated a travel time for each arc and based of the probability of the optimistic and pessimistic cases. Then the sum-product was taken of the arc lengths determined by the @Risk tool and binomial cells that took on the value of 1 if that arc was used in the route. This sum-product was set up as the output cell for the @Risk Simulation as it gave the total travel time that it would take a vehicle to travel that path for the current values of the arcs travel times. For further comparison, the same networks and probabilities were used to calculate total travel times in the exact same way, but this time the @Risk software used a uniform distribution based on the high and low values for each arcs travel time. Using the @Risk software allowed for each combination of network and probability distribution to be ran for 10,000 replications, and the results are shown below in Tables 7-10.

Network 1

Uncertainty With Discrete Probability Used For Expected Values		
Travel Time Data	Using Route Suggested When There was No Uncertainty	Using Route Suggested When Expected Values Were Used
Average Over Trials (Std Dev)	12.4 (2.5)	11.9 (.76)
Max Over Trials	17	15

Table 7

Uncertainty With Uniform Probability Used For Expected Values		
Travel Time Data	Using Route Suggested When There was No Uncertainty	Using Route Suggested When Expected Values Were Used
Average Over Trials (Std Dev)	12.5 (1.5)	13 (.6)
Max Over Trials	16.6	14.8

Table 8

Network 2

Uncertainty With Discrete Probability Used For Expected Values		
Travel Time Data	Using Route Suggested When There was No Uncertainty	Using Route Suggested When Expected Values Were Used
Average Over Trials (Std Dev)	19 (2.1)	18.1 (1.7)
Max Over Trials	22	21

Table 9

Uncertainty With Uniform Probability Used For Expected Values		
Travel Time Data	Using Route Suggested When There was No Uncertainty	Using Route Suggested When Expected Values Were Used
Average Over Trials (Std Dev)	18 (1.3)	18 (1)
Max Over Trials	21.7	20.9

Table 10

When a discrete probability is used, the route that is considered optimal when the expect values are used to for travel times along the arcs not only minimized the time of arrival at the last node in the tour, but also had the lower maximum travel time encountered when there was uncertainty present. This is true for both networks analysed as in network 1, the average travel time was .5 units less and the maximum travel time over the trials is 2 units lower when using the expected value optimal route, and in network 2 the average

travel time was .9 units less and the maximum travel time over the trials is 1 unit lower when using the expected value optimal route. In the simulations where the travel time could equally be any value within the bounds of the optimistic and pessimistic value (uniform distribution), a different result occurred.

In the first network, the route that was optimal when the expected value of the uncertain parameter was used still had the lower maximum value for the time arrived at the last node, but the route that was optimal when there was no uncertainty in the model had the lower average time of arrival at the last node in the tour. In the second network, the route that was optimal when using the expected values still had the lower maximum travel time, but the average travel time was identical regardless of which route was taken. Although I expected the case which took the expected values into account to be better in all areas, the results from the simulation reflect positively on the idea of using the expected values in route calculation. Looking at network 2, the expected value route performed comparatively to the originally optimal route when it came to average travel time, while recording a lower maximum travel time in both cases. Although the same is not true for network 1, the results seem to support the idea that using the expected values during route formulation is still a good idea. In the case where uniform probabilities were used, by using the optimal route based off the expected values, you only sacrifice half a unit of time (.5) in order to reduce the maximum travel time that could be incurred by almost two time units (1.8). I believe in a disaster situation this trade off would be favorable as it is a small sacrifice to make in order to greatly protect the route against a higher worst case scenario value.

References

[1] Burcu Adivar and Ali Mert. International disaster relief planning with fuzzy credibility. *Fuzzy Optim Decis Making*, 9:413–433, 2010.

[2] Serhan Duran, Marco A. Gutierrez, and Pinar Keskinocak. Pre-positioning of emergency items for care international. *Interfaces*, 41(3):223–237, 2011.

[3] Rongbing Huang, Seokjin Kim, and Mozart B.C. Menezes. Facility location for large-scale emergencies. *Annual Operations Research*, 181:271–286, 2010.

[4] Pamela C. Nolz, Frederic Semet, and Karl F. Doerner. Risk approaches for delivering disaster relief supplies. *OR Spectrum*, 33:543–569, 2011.

[5] Yoshinori Suzuki. Disaster-relief logistics with limited fuel supply. *Journal of Business Logistics*, 33(2):145–147, 2012.

[6] Paolo Toth and Daniele Vigo, editors. *The Vehicle Routing Problem*. SIAM.

[7] E. van Wyk, W.L. Bean, and V.S.S. Yadavalli. Modelling of uncertainty in minimising the cost of inventory for disaster relief. *South African Journal of Industrial Engineering*, 22(1):1–11, 2011.