§24. General and Accurate Weighting Scheme for Collisional δf Particle Simulation

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Let us solve linearized drift kinetic equation

$$\frac{D}{Dt}f_1 = -\vec{v_d}\cdot\nabla f_0 + C(f_0, f_1) \qquad (1)$$

from the dynamics of a finite number of particles (markers), where the notation $D/Dt(f)$ denotes

$$\frac{D}{Dt}f = \frac{\partial f}{\partial t} + (\vec{v_\parallel} + \vec{v_d})\cdot\nabla f - C(f, f_0). \qquad (2)$$

The distribution function f_1 is represented as

$$f_1(\vec{x},\vec{v},t) = \sum_j w_j\delta(\vec{x}-\vec{x}_j(t))\delta(\vec{v}-\vec{v}_j(t)). \qquad (3)$$

The particle weights w are calculated from

$$\dot{w} = \frac{1}{g}\left[-\int wS_M dw - \vec{v_d}\cdot\nabla f_0 + C(f_0, f_1)\right], \qquad (4)$$

where S_M is the marker source used to control marker population during simulation and g is marker density[1]. The calculation of particle weight takes a central position in a δf simulation. It is demonstrated that valid results essentially rely on the correct evaluation of marker density g in weight calculation. Previous weighting schemes including the nonlinear weighting scheme[2] employ an assumed g in weight equation for advancing particle weights. Such a scheme is ineffective or inaccurate to solve drift kinetic equation because of a severe constraint the real marker distribution must be consistent with the assumption during a simulation. Instead using an approximation of g for advancing particle weights, we solve g directly from its kinetic equation

$$\frac{D}{Dt}g = \int S_M dw. \qquad (5)$$

Setting $g = f_0 + g_1$, we obtain

$$\frac{D}{Dt}g_1 = -\vec{v_d}\cdot\nabla f_0 + \int S_M dw. \qquad (6)$$

We employ a new weight function ω and a marker distribution function $G_M(\vec{x},\vec{v},\omega,t)$, and then solve Eq.(6) using the idea of δf method. Let G_M obey the kinetic equation

$$\frac{D}{Dt}G_M + \frac{\partial}{\partial\omega}(\dot{\omega}G_M) = \Omega_M(\vec{x},\vec{v},\omega,t) \qquad (7)$$

with $G_M(t=0) = f_0\delta(\omega)$, and relate to g_1 through

$$g_1(\vec{x},\vec{v},t) = \int \omega G_M d\omega. \qquad (8)$$

Here a source Ω_M, like S_M, is introduced to control marker population. From the requirement that Eqs.(6)-(8) are consistent, the equation for ω is determined as

$$\dot{\omega} = \frac{1}{h}\left[-\int \omega\Omega_M d\omega - \vec{v_d}\cdot\nabla f_0 + \int S_M dw\right], \qquad (9)$$

where distribution function $h(\vec{x},\vec{v},t)$ satisfies

$$\frac{D}{Dt}h = \int \Omega_M d\omega. \qquad (10)$$

Taking

$$\int \Omega_M d\omega = \int S_M dw. \qquad (11)$$

Then Eqs.(10) and (5) become the same. We have

$$h(\vec{x},\vec{v},t) = g(\vec{x},\vec{v},t). \qquad (12)$$

For a marker of index j, we have

$$g_j = g_{0j} + \omega_j h_j \qquad (13)$$

which gives

$$g_j = \frac{1}{1-\omega_j}f_{0j}. \qquad (14)$$

Substituting Eq.(14) into Eqs.(4) and (9), we obtain weight equations for a marker as follows

$$\dot{w} = \frac{1-\omega}{f_0}\left[-\int wS_M dw - \vec{v_d}\cdot\nabla f_0 + C(f_0, f_1)\right], \qquad (15)$$

$$\dot{\omega} = \frac{1-\omega}{f_0}\left[-\int \omega\Omega_M d\omega - \vec{v_d}\cdot\nabla f_0 + \int S_M dw\right]. \qquad (16)$$

Now each simulation particle is assigned two weights rather than one, the second weight ω is introduced to effectively evaluate g (exactly saying, g_1), and we can solve Eqs.(1) and (6) simultaneously using one set of markers. Equations (15) and (16) represent a general and accurate weighting scheme. For practical application it is convenient to take $S_M = \nu(t)s(\vec{r})f_0\delta(w)$ and $\Omega_M = \nu(t)s(\vec{r})f_0\delta(\omega)$. This choice means that new Maxwellian markers with $w = \omega = 0$ are added in terms of rate $\nu(t)$ and spatial distribution $s(\vec{r})$. Then the weight equations are simplified to

$$\dot{w} = \frac{1-\omega}{f_0}[-\vec{v_d}\cdot\nabla f_0 + C(f_0, f_1)], \qquad (17)$$

$$\dot{\omega} = \frac{1-\omega}{f_0}[-\vec{v_d}\cdot\nabla f_0 + \nu s f_0]. \qquad (18)$$

References

1) Chen,Y. and White,R.B. Phys. Plasmas 4 (1997) 3591.
2) Lin,Z., Tang,W.M. and Lee,W.W., Phys. Plasmas 2 (1995) 2975.