§9. Rotational Equilibrium of Nonneutral Plasma

Tomita, Y., Yasaka, Y. (Faculty of Engineering, Kyoto Univ.)

A double-cusp type direct energy converter was proposed from thermal energy out of confined plasma to electricity.1) In this conversion system charge separation is necessary to recover ion kinetic energy. The equilibrium of nonneutral plasma (pure ion plasma) was studied.

The thermal equilibrium ion distribution in the axisymmetry system is given by 2)

$$f_i = \alpha_i \left(\frac{m_i}{2\pi T_i}\right)^{3/2} \exp\left(-\frac{H_i - \omega_i P_{\theta i} - u_i P_{zi}}{T_i}\right) \quad (1)$$

where

$$H_i = \frac{m_i v^2}{2} + q_i \phi(r), \quad (2)$$

$$P_{\theta i} = m_i r v_\theta + q_i r A_\theta(r), \quad (3)$$

$$P_{zi} = m_i v_z \quad (4)$$

are the energy, canonical angular momentum, and canonical momentum, respectively. The quantities $\phi(r)$ and $A_\theta(r)$ are the electric potential and the azimuthal component of the vector potential, respectively. And the quantities α_i, ω_i and u_i are constants. For a uniform axial magnetic field B_z , the vector potential is given by $A_\theta(r) = rB_z/2$. By using this vector potential, ion distribution function can be written as

$$f_i(r,\vec{v}) = n_i(r)\left(\frac{m_i}{2\pi T_i}\right)^{3/2} \exp\left[\frac{-m_i}{2T_i}(\vec{v} - r\omega_i\hat{\theta} - u_i\hat{z})^2\right], \quad (5)$$

$$n_i(r) = \int_{-\infty}^{\infty} d\vec{v} f_i = n_i(0)\exp[-\xi_i(r)] , \quad (6)$$

$$n_i(0) = \alpha_i \exp[m_i u_i^2/2T_i], \quad (7)$$

$$\xi_i(r) = \frac{1}{T_i}\left[q_i\phi(r) - \frac{m_i}{2}r^2\omega_i^2 - \frac{q_i\omega_i B_z}{2}r^2\right] \quad (8)$$

From the average velocity

$$\langle v_\theta \rangle(r) = \int_{-\infty}^{\infty} v_\theta f_i d\vec{v} / n_i(r) = r\omega_i, \quad (9)$$

$$\langle v_z \rangle(r) = \int_{-\infty}^{\infty} v_z f_i d\vec{v} / n_i(r) = u_i, \quad (10)$$

one can see the plasma rotates as rigid rotor with angular frequency ω_i and translates to axial direction with speed . The ion density distribution is determined by three potentials: the electric potential, the centrifugal potential, and the potential associated with the electric field induced by rotation through the magnetic field. The density should approach to 0 according to infinity radius, i.e.

$$r \to \infty : \ \xi_i(r) \cong -\frac{m_i\omega_i r^2}{2T_i}\left(\omega_i + \frac{q_i B_z}{m_i}\right) > 0. \quad (11)$$

This gives the rotation direction to that of magnetic field:

$$\omega_i\left(\omega_i + \frac{Z_i e B_z}{m_i}\right) < 0. \quad (12)$$

The electric potential is determined by Poisson's equation

$$\frac{1}{r}\frac{d}{dr}r\frac{d\phi}{dr} = -\frac{1}{\varepsilon_0}\sum_j q_j n_j(r) = -\frac{1}{\varepsilon_0} q_i n_i(0)\exp[-\xi_i(r)] , \quad (13)$$

and the boundary conditions $\phi = d\phi/dr = 0$ at $r = 0$. The distributions of density and electric potential with rotational equilibrium are shown in Fig.1, where Ω_{ci} and ω_{pi} are the ion cyclotron and plasma frequencies, respectively. The smaller value of Ω_{ci}/ω_{pi} does not give the equilibrium configuration. The effects of electron to pure ion plasma should be studied.

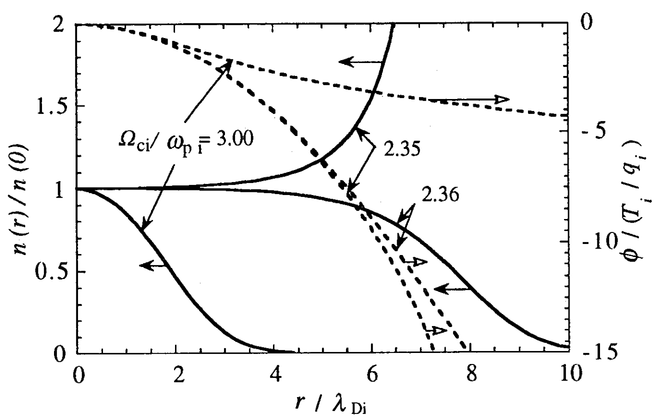

Fig. 1. Equilibrium distributions of ion density and electric potential with rigid rotation. The quantities Ω_{ci}, ω_{pi}, and λ_{Di} are the ion cyclotron, plasma frequencies and ion Debye length, respectively.

Reference

1) H.Momota, et al., Proc. 14th Int. Conf. Plasma Physics and Controlled Nucl. Fusion Research, Wüerzburg, Germany, September 1992, Vol.3 p.319, IAEA (1993).
2) R.C.Davidson, Theory of Nonneutral Plasmas (Benjamin, Reading, Mass., 1974).