§10. Electrostatic Potential due to Induced Charge of Spherical Dust Particle on Plasma-Facing Wall in Non-uniform Electric Field

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A spherical conducting dust particle with a radius R_d is attached on an infinitely extended conducting plane wall. The local electrostatic potential is composed of the sum of the external one ϕ_{ex} and the one ϕ_{in} due to induced charges on the conducting dust particle.

$$\phi(r,z)=\phi_{ex}(z)+\phi_{in}(r,z) \qquad (1)$$

where (r,z) is the conventional cylindrical coordinate, where the origin is located at the contacting point of the spherical dust to the plane wall (Fig.1).

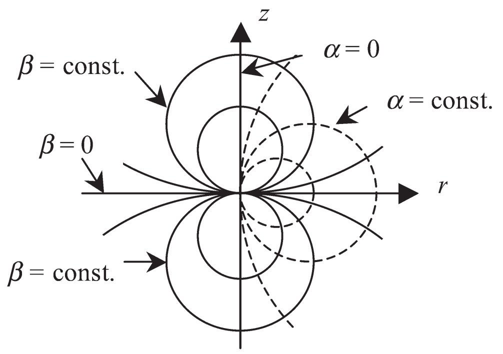

Fig.1 The cylindrical coordinate (r,z) and the bipolar coordinate (α,β). The infinitely extended conducting plane wall and the surface of the spherical dust are indicated by $\beta = 0$ and 1/2, respectively.

The potential ϕ_{in} due to the induced charge satisfies the Laplace equation:

$$\frac{\partial}{\partial\alpha}\left(\frac{\alpha}{\alpha^2+\beta^2}\frac{\partial\phi_{in}}{\partial\alpha}\right)+\frac{\partial}{\partial\beta}\left(\frac{\alpha}{\alpha^2+\beta^2}\frac{\partial\phi_{in}}{\partial\beta}\right)=0 \qquad (2)$$

in the bipolar coordinate (α, β) [5]. The relation between the cylindrical coordinate (r,z) and the bipolar coordinate (α,β) is

$$z+ir=\frac{iR_d}{\alpha+i\beta}, \qquad (3)$$

where i denotes the imaginary unit. The general solution of the Laplace equation (2), which satisfies the condition ϕ_{in} = 0 at $\beta = 0$, is given by

$$\phi_{in}(\alpha,\beta)=\sqrt{\alpha^2+\beta^2}\sum_{n=1}^{\infty}c_n I_{\lambda n}(\alpha,\beta), \qquad (4)$$

and

$$I_{\lambda n}(\alpha,\beta)\equiv\int_0^{\infty}d\lambda\lambda^n e^{-\beta_0\lambda}\frac{\sinh(\beta\lambda)}{\sinh(\beta_0\lambda)}J_0(\lambda\alpha) \qquad (5)$$

where J_0 is the first kind Bessel function of the 0-th order and the coefficients c_n's are determined by the boundary condition at the surface of the spherical dust. In this study the externally applied potential is approximated to be non-uniform toward the normal direction to the plane wall, i.e. z, which is expressed in (α,β):

$$\phi_{ex}(\alpha,\beta)=\sum_{k=0}^{k\max}h_k z^k=\sum_{k=0}^{k\max}h_k\left(\frac{R_d\beta}{\alpha^2+\beta^2}\right)^k \qquad (6)$$

where the coefficient h_0 corresponds to the biased wall potential ϕ_w. The total electrostatic potential $\phi(\alpha,\beta)$ is obtained as a combination of the externally applied potential ϕ_{ex} and the induced potential ϕ_{in}. In case of $k_{\max}$ = 4,

$$\begin{aligned}\phi(\alpha,\beta)&=\phi_{ex}(\alpha,\beta)+\phi_{in}(\alpha,\beta)\\&=\phi_w+\sqrt{\alpha^2+\beta^2}[(h_1R_d+\frac{h_2R_d^2}{3\beta}+\frac{h_3R_d^3}{5\beta^2}+\\&\frac{h_4R_d^4}{7\beta^3})I_1(\alpha,\beta)+(\frac{h_2R_d^2}{3}+\frac{h_3R_d^3}{5\beta}+\frac{h_4R_d^4}{7\beta^2})I_2(\alpha,\beta)\\&+(\frac{h_3R_d^3}{15}+\frac{2h_4R_d^4}{35\beta})I_3(\alpha,\beta)+\frac{h_4R_d^4}{105}I_4(\alpha,\beta)\\&+\sqrt{\alpha^2+\beta^2}\sum_{k=1}^{\infty}c_kI_{\lambda k}(\alpha,\beta)\end{aligned} \qquad (7)$$

where the first and second terms of the RHS of eq.(7) are the external potential and the last term corresponds the induced potential. From the boundary condition at the surface of the spherical dust $\beta = \beta_0 = 1/2$, the coefficients c_k's in Eq.(4) are expressed by the known quantities. Finally we obtain the total local potential consisting of the external and induced ones for the case $k_{\max} = 4$:

$$\begin{aligned}\phi(\alpha,\beta)&=\phi_{ex}(\alpha,\beta)+\phi_{in}(\alpha,\beta)\\&=\phi_w+\sqrt{\alpha^2+\beta^2}\{h_1R_dI_{s1}+\\&+\frac{h_2R_d^2}{3}(\frac{\beta_0-\beta}{\beta_0\beta}I_1+\frac{I_{s1}}{\beta_0}+I_{s2})\\&+\frac{h_3R_d^3}{5}(\frac{\beta_0^2-\beta^2}{\beta_0^2\beta^2}I_1+\frac{I_{s1}}{\beta_0^2}+\frac{\beta_0-\beta}{\beta_0\beta}I_2+\frac{I_{s2}}{\beta_0}+\frac{I_{s3}}{3})\\&+\frac{2I_{s3}}{5\beta_0}+\frac{I_{s4}}{15}]\}\end{aligned}$$

Here the function I_{sn} is defines as

$$I_{sn}(\alpha,\beta)\equiv\int_0^{\infty}d\lambda\lambda^n\frac{\sinh[(\beta_0-\beta)\lambda]}{\sinh(\beta_0\lambda)}J_0(\lambda\alpha).$$

These expressions can be useful to obtain the induced charge on the dust surface.