§17. Interference Effects in the Decay of Resonance States in Three-Body Coulomb Systems

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We have analyzed the lowest $^1S^e$ resonance state in a family of symmetric three-body Coulomb systems with charges 1, 1, and -1 and masses *M*, *M*, and 1 as a function of the parameter *M* which is the mass-ratio for the particles[1]. Accurate calculations (by Siegert pseudostate method) of the resonance position E(*M*) and width Γ(*M*) in the range 0 < M < 30 are shown in Fig.1.

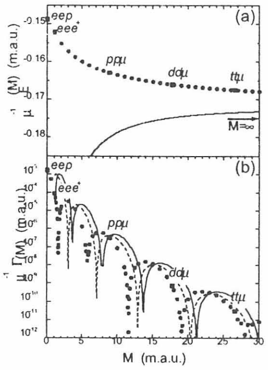

Fig.1.Bold circles and squares: accurate results for the resonance position E(*M*) and width Γ(*M*) calculated by the Siegert pseudostate method. Solid and dashed curves: the results obtained by perturbation analysis of the Born-Oppenheimer model.

This range includes several real three-body Coulomb systems *eep*, eee^+, *ppμ*, *ddμ*, and *ttμ* for which we have obtained the most accurate estimates of the resonance parameters.

System	M	-E(*M*)	Γ(*M*)
eep	$0.54461701 \cdot 10^{-3}$	0.148695(1) au	$0.1731(1) \cdot 10^{-2}$ au
eee^+	1	0.0760304(1) au	$0.4304(1) \cdot 10^{-4}$ au
ppμ	8.8802445	0.146404(1) μu	$0.304(1) \cdot 10^{-6}$ μu
ddμ	17.751675	0.157099(1) μu	$0.69(1) \cdot 10^{-9}$ μu
ttμ	26.584939	0.161370(1) μu	$0.3(1) \cdot 10^{-10}$ μu

The principal finding of these calculations is that Γ(*M*) oscillates as a function of *M*, which reveals an interference mechanism in the resonance decay dynamics. The perturbation Fermi-Fano-Feshbach analysis of a simplified Born-Oppenheimer model obtained from the three-body Coulomb problem in the limit $M \to \infty$ extends the range of the mass-ratio up to *M*=300. These results are shown in Fig.2.

This analysis confirms that Γ(*M*) continues to oscillate with an increasing period and decreasing envelope as *M* grows. Simultaneously it suggests that the mechanism of the oscillations could be interpreted in terms of semiclassical theory. The key role in such interpretation belongs to what is known as the Demkov construction. Decay of a resonance in this approach amounts to passing from the initial Riemann sheet of the adiabatic potential energy corresponding to the closed channel to a lower sheet corresponding to the open channel around a branch point connecting the sheets. Then the oscillations of Γ(*M*) can be interpreted as a result of interference between two paths of the resonance decay, one of which goes directly to fragmentation region, while the other one first passes through the turning point on the lower sheet. In other words, the oscillations of Γ(*M*) are a manifestation of the Stueckelberg phase well known from analysis of various two-state models. The semiclassical analyses gives the following expression for the resonance position for large values of *M*:

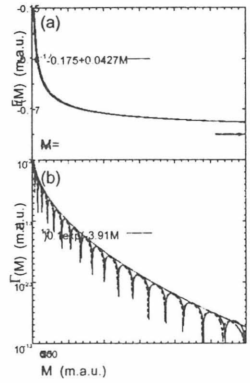

Fig.2. Solid and dashed curves: results for the resonance position E(*M*) and width Γ(*M*) obtained from the Born-Oppenheimer model, i.e. the same as the corresponding curves in Fig.1,but for a wider interval of the mass-ratio *M*. Dotted curves: the semiclassical results obtained (a) from Eq.(1) and (b) by fitting the envelope of the Born-Oppenheimer results by that dictated by Eq.(2) .

$$E(M)_{M\to\infty} = -0.175049036 + \frac{0.042706071}{\sqrt{M}} \quad (1)$$

This nicely reproduces the accurate numerical results (Fig.2a.) The functional form of the dependence on *M* of the resonance width obtained on the basis of the semiclassical theory is given by

$$\Gamma(M) = a(M)e^{-\alpha(M)}\sin^2\varphi(M) \quad (2)$$

where α(*M*) is the Massey parameter and ϕ(*M*) is the Stueckelberg phase. The envelope of the function Γ(*M*) and the period of its oscillations given by this equation agree excellently with the present numerical results. This supports the semiclassical interpretation and warrants more detailed study of the multivalued adiabatic potential energy for the three-body Coulomb problem and further development of the Demkov's construction in the framework of the hyperspherical method.

Reference:

[1] O.I.Tolstikhin, I.Yu.Tolstikhina, and C.Namba.; Phys. Rev. A. **60**, 4673-92 (1999).