§2. Numerical Matching Scheme for MHD Evolution Equation

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A new matching scheme for linear MHD stability analysis is proposed in a form that the numerical implementation is tractable. The scheme divides the plasma region into outer regions and inner layers, as in the conventional matching method. However the outer regions do not contain any rational surface as their terminal points; an inner layer contains a rational surface as an interior point. The Newcomb equation is therefore regular in the outer regions. The MHD equation in the layers is solved as an evolution equation in time. The matching conditions are derived from the conditions that the radial component of the solution in the layer be smoothly connected to those in the outer regions at the terminal points. The proposed scheme is applied to the linear ideal MHD equation in a cylindrical configuration, and is proved to be effective from the view point of numerical scheme.

We employ the linear ideal MHD equation

$$\rho \partial_t^2 \xi(r,t) = F[\xi(r,t)] \qquad (1)$$

in the cylindrical coordinate system (r, θ, z); the infinitesimal displacement vector of plasma is assumed to be

$$\xi(r,t)\exp(im\theta - ikz),$$

ρ the density of plasma, and F[ξ] the force operator. We assume the fixed boundary condition $y(a, t) = 0$ for the radial component of the vector ξ. We solve Eq. (1) around the inner layer (r_L, r_R), (we call them matching points), by using the full implicit scheme

$$\rho \xi^{n+1} - (\Delta t)^2 F[\xi^{n+1}] = S_{in}[\xi] \quad (2)$$

$$S_{in}[\xi] = \rho(2\xi^n - \xi^{n-1}) \quad (3)$$

where

$$\xi^n(r) = \xi(r, n\Delta t)$$

and Δt is the time step in the scheme. Equation (2) reduces to an inhomogeneous linear differential equation of second order in the radial component $y^{n+1}(r)$, which should be solved with appropriate boundary conditions. In the outer regions

$$(0, r_L), \quad (r_R, a)$$

far from the rational surface, where the inertia terms are regarded as small, we solve F[ξ] = 0, which reduces to the Newcomb equation. The solutions of the Newcomb equation are given by

$$y_L(r,t) = \xi_L(t) G_L(r),\ 0 \le r \le r_L$$
$$y_R(r,t) = \xi_R(t) G_R(t),\ r_R \le r \le a$$

where $G_p(r)$ is the solution of the Newcomb equation that satisfies

$$G_p(r_p) = 1 \quad (p = L, R)$$

and the appropriate boundary condition at $r = 0$ or $r = a$; $\xi_p(t)$ are the values of the solution at the matching points, which are unknown. We can construct the solutions of Eq. (1) and the Newcomb equation in such a way that the radial components of the solutions take at the matching points the same values ξ_p^{n+1}. We moreover impose on the solutions as the matching conditions that their derivatives at the points are equal to each other. Those give us a linear equation on ξ_p^{n+1}: They are easily solved; hence the matching problem can be solved numerically. Numerical experiments on the classical $m = 1$ internal kink mode confirm the validity of the proposed scheme[1].

[1] Kagei, Y., Tokuda, S., submitted to Plasma Phys. Fusion Res.