§9. Effect of Finite Orbit Size of Ripple Trapped Particles on Neoclassical Transport in Helical Torus

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The theory on transport process of the plasma in the toroidal configuration is based on the assumption that the fundamental plasma properties are described by several averaged quantities such as number density, temperature, and so on, which are considered to be functions of magnetic surfaces. This assumption implies that the plasma particles stay on the magnetic surface without collision, or more generally, the deviation of the orbit from the magnetic surface is much smaller than the characteristic length of the variation. In some experimental situations, however, the orbit of some particles deviates significantly from magnetic surfaces; the inclusion of such effect into the transport theory remains as an important issue of the theory.

In the collisionless limit, the distribution function is constant along the particle orbit in the phase space. Therefore the distribution function may be considered function of constant of motion, i.e. energy, magnetic moment, and the longitudinal adiabatic invariant. However, the statement that in the limit of low collisionality the distribution function is described only by the constant of motion is not correct in the toroidal plasma

In the toroidal configuration there are several kinds of particles with orbit having the different topology. Since the collisional effects are described by the second order differential operator in the velocity space, continuity of the distribution function and its derivative with respect to the energy and magnetic moment is the requisite. The both requirements cannot be satisfied at the boundary of the different orbit topology. This means that, even if the collision frequency is small, the distribution function is not constant along the particle orbit in the certain part of the phase space, where the collisionality is effectively enhanced.

We consider the helical torus with toroidal period N and the rotational transform per period is assumed to be small: $\iota/N \ll 1$. The motion of charged particles in such systems are described by using the longitudinal adiabatic invariant J. The representation of the adiabatic invariant differs for passing particles and ripple-trapped particles, due to the difference of the orbit topology.

As we consider the small Larmor radius case, the motion of the passing particles is regarded as lying upon the magnetic surface. The distribution function of the passing particles is essentially local Maxwellian, the density and the temperature are functions of the magnetic surface.

The characteristic time for the ripple-trapped particles is longer than that of passing particles. When the excursion time of the trapped particles ω_T^{-1} is much longer than the effective collision time ν_{eff}^{-1}, the effect of the finite orbit deviation is not essential, and the flux proportional to ν_{eff}^{-1} is found. The opposite limit is the most interesting case, and it is the main object of this study.

The main body of the ripple-trapped particles is considered as collisionless, and the distribution function f^T is expanded as

$$f^T = F^{(0)} + (\nu/\omega_T)F^{(1)} + \cdots \tag{1}$$

where $F^{(0)} = F^{(0)}(J,\lambda)$, and $\lambda \equiv \mu B_0/E$ is the pitch angle.

Choosing an action variable $H = H(J,\lambda)$, we expand the distribution function as

$$F^{(0)}(H,\lambda) = F_0^{(0)}(H) + F_1^{(0)}(H,\lambda) + \cdots \tag{2}$$

with respect to some small parameter. Such an expansion may not be validated unless the action variable H is properly chosen. Since the orbit has different topology depending on the presence of transition point, the choice of the variable H has to be made separately for each case.

Suppose the orbit curve $J(\psi,\theta;\lambda) = \mathrm{const}.$ crosses the curve at the transition points $(\psi_t, \pm\theta_t)$. At the points, the distribution function of ripple-trapped particle is equal to that of passing particles. If we choose as

$$H = \psi_t(J,\lambda), \tag{3}$$

then we have

$$F_0^{(0)}(H) = f_0(\psi_t), \tag{4}$$

When the collisionless orbit has no transition point, the action variable H is chosen so that the coefficients before the derivative with respect to H in the collision operator is small. If the curve H = constant crosses the boundary curve with the region with transition points, the function $F_0^{(0)}(H)$ is determined from the continuity condition at the boundary.

The transport flux for the mono-energetic particles is obtained by the integral

$$\Gamma = \int d\theta \int d\lambda F^{(1)} \frac{1}{e}\frac{\partial J}{\partial\theta}. \tag{5}$$

The integral is carried over the fixed magnetic surface. If we consider the case that the helical ripple is small, the part of the ripple-trapped particle with transition gives the main contribution for the transport flux. The integration with respect to θ is changed to that with respect to the action variable H, which is essentially the magnetic surfaces.

The transport flux across the magnetic flux can be obtained in the integral form as

$$\Gamma = \int_{\psi_L}^{\psi} d\psi \mathcal{V}_1(\psi,\psi_*)\frac{\partial f_0}{\partial\psi_*} + \int_{\psi_L}^{\psi} d\psi \mathcal{V}_2(\psi,\psi_*)\frac{\partial^2 f_0}{\partial\psi_*^2}. \tag{6}$$

Thus, the transport flux cannot be expressed by single coefficient.