§34. Modified K-dV Equation for Nonlinear Magnetosonic Waves in a Multi-Ion Plasma

Todoroki, J., Sanuki, H.

Nonlinear magnetosonic waves have been extensively studied because they play an important role in particle acceleration and heating of plasmas. Recently, the nonlinear behavior of both low -and high -frequency magnetosonic waves has been discussed by Toida and Ohsawa(ref.1), who showed that these waves are described by K-dV equation, although the dispersion branch of high frequency mode has a finite cut-off frequency. We should note that we have to discuss very carefully the scaling and ordering in derivation of a nonlinear wave equation for this mode under the influence of finite cut-off frequency.

Paying attention to this situation and applying a proper ordering and scaling with respect to amplitude and mass ratio involved in the present set of equations, we derive a nonlinear magnetosonic wave equation , which includes effects of the finite cut-off frequency.

We here study a magnetosonic waves propagating in the direction(x) perpendicular to a magnetic field(z) on the basis of following fluid equations with two ion-species:

$$\frac{\partial n_j}{\partial t}+\frac{\partial}{\partial x}\left(n_j v_{xj}\right)=0,$$

$$\frac{\partial v_{xj}}{\partial t}+v_{xj}\frac{\partial v_{xj}}{\partial x}=\frac{q_j}{m_j}\left(E_x+\frac{1}{c}v_{yj}B_z\right),$$

$$\frac{\partial v_{yj}}{\partial t}+v_{xj}\frac{\partial v_{yj}}{\partial x}=\frac{q_j}{m_j}\left(E_y-\frac{1}{c}v_{xj}B_z\right),$$

$$\frac{\partial B_z}{\partial t}=-c\frac{\partial E_y}{\partial x},\quad \frac{\partial B_z}{\partial x}=-\frac{4\pi}{c}\sum_j q_j n_j v_{yj},$$

$$\sum_j q_j n_j v_{xj}=0.$$

In the following derivation, we apply the assumptions,

$$\frac{\omega_{pe}^2}{c^2}\gg k^2\gg\frac{\omega_{pi}^2}{c^2},$$

$$\frac{m_e}{m_i}\ll 1,$$

Then , using the conventional reductive perturbation method and introducing the following stretched variables and ordering

$$\xi=\varepsilon^{1/2}(x-Vt),\quad \tau=\varepsilon^{3/2}t.$$

$$\frac{m_e}{m_i}\sim\varepsilon^2,$$

we finally obtain a modified K-dV equation in a form

$$\frac{\partial}{\partial\xi}\left\{\frac{\partial u}{\partial\tau}+\beta u\frac{\partial u}{\partial\xi}+\gamma\frac{\partial^3 u}{\partial\xi^3}\right\}-\delta u=0.$$

where

$$\beta=\frac{3}{2}\frac{\omega_{pe}^2(\omega_{pa}^2\Omega_a+\omega_{pb}^2\Omega_b)}{\Omega_e(\omega_{pa}^2+\omega_{pb}^2)^2},\quad \gamma=\frac{c^2V}{2\omega_{pe}^2},$$

$$\delta=\frac{V}{2c^2}\frac{\omega_{pa}^2\omega_{pb}^2(\Omega_a-\Omega_b)^2}{(\omega_{pa}^2+\omega_{pb}^2)^2}\left\{\frac{\omega_{pa}^2}{\Omega_a^2}+\frac{\omega_{pb}^2}{\Omega_b^2}\right\}.$$

If we neglect the δ —term, this equation reduces to the equation derived by Toida and Ohsawa. We note that the characteristics of this equation sensitively depends on the competing effect between γ -and δ -terms and is essentially different from the previous results(ref.1). Detailed discussions is now under investigation.

1) M. Toida and Y. Ohsawa: in Report of Plasma Science Center, Nagoya University PSC-33 Nov. (1993).