§15. Localized Stability Criterion and Double Adiabatic Theory

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Magnetohydrodynamic (MHD) stability is regarded as one of the most important issues in magnetic confinement of fusion plasma. In theoretical investigation of the MHD stability of helical torus plasma, the Mercier stability criterion for the localized mode is used as a convenient measure of the instability.

In the LHD experiment plasma confinement improves due to inward shifts of the plasma, as is expected theoretically; and much effort has been devoted to pursuing further improvements by achieving larger inward plasma shifts. Current theory predicts strong instability in cases of inward plasma shifts of over 15–20 cm [1]. Experimentally, however, inward shifts of even 30–40 cm can be made without creating severe instability [2].

It is of great importance to understand the origins of this conflict between theory and experiment. The predicted instability is so strong that it is unplausible to explain by changes in pressure profiles or toroidal current profiles. We propose here a plasma model based on double adiabaticity as a possible explanation for stability of LHD plasma beyond the Mercier criterion.

The collisionless plasma placed in the strong magnetic field obeys the double adiabatic law [3]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_{//}B^2}{\rho^3}\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\perp}{\rho B}\right) = 0. \quad (1)$$

Here, $p_{//}$ and $p_\perp$ stand for the pressure parallel and perpendicular to the magnetic field, and ρ is the mass density. Conventional MHD theory assumes the single adiabatic law, which implies frequent collisions mixing the freedoms of the particle motion in different directions. When the plasma temperature rises, the plasma becomes more collision-free, and the descriptions of motion based on double adiabatic laws may be more appropriate for such plasmas. Here, we assume that the equilibrium plasma pressure is isotropic $p_{//} = p_\perp = p(\psi)$; only the perturbed pressure may be anisotropic.

In conventional MHD with single adiabaticity, the marginal stability is characterized as $\nabla\cdot\boldsymbol{\xi} = 0$; the instability decouples with the sound wave. This is not true in the case of double adiabaticity, and the coupling with the sound wave may significantly stabilize the interchange mode.

The localized mode in the toroidal geometry is investigated. In the vicinity of the rational surface with $\iota_0 \equiv \iota(\psi_r) = n_0/m_0$, the stretched coordinates are introduced.

$$x = \varepsilon^{-1}(\psi - \psi_r),\ y = \theta - \theta_0 - \iota_0\phi,\ \zeta = \phi. \quad (2)$$

By expanding the plasma displacement with respect to the small parameter ε, the plasma potential energy is minimized step by step. The plasma pressure is assumed as small as $p \sim O(\varepsilon^2)$. The minimization with respect to the component perpendicular to the magnetic field can be carried out similar to the single adiabatic case. The minimization with respect to the displacement parallel to the magnetic creates difficulty, as the ripple of the magnetic field strength along the magnetic line of force causes coupling with the sound wave, making it necessary to solve a second order differential equation. If we assume a small magnetic ripple, i.e. $h \sim \delta \ll 1$ where $h \equiv \boldsymbol{B}\cdot\nabla\ln B$, the second order differential equation can be solved by approximation, and the expression for the plasma potential energy density is obtained as

$$\overline{W}^{(0)} \propto s^2x^2\left|\frac{\partial\xi_\psi^{(0)}}{\partial x}\right|^2 - D\left|\xi_\psi^{(0)}\right|^2 + C_{DA}\left|\xi_s^{(0)}\right|^2, \quad (3)$$

where $s \equiv \iota_0'$ is the magnetic shear, D is the coefficient appearing in the Mercier criterion ($\frac{1}{4}s^2 - D \geq 0$), and

$$C_{DA} = \frac{5}{3}p\left\langle\frac{B^2}{|\nabla\psi|^2}\right\rangle\left\{\left\langle\kappa_s^{\,2}\right\rangle - \frac{\left\langle\kappa_s h\right\rangle^2}{\left\langle h^2\right\rangle}\right\}. \quad (4)$$

Here κ_s is the geodesic curvature

$$\kappa_s = \frac{\boldsymbol{B}\times\nabla\psi}{2B^4}\cdot\nabla B^2, \quad (5)$$

with the bracket indicating the average along the closed magnetic line of force. Because the expression (4) is always positive, the last term in eq. (3), which is not in the single adiabaticity MHD, act as a stabilizer. Due to this stabilization term, the size of localization around the rational surface becomes bounded.

By using the potential energy and kinetic energy terms, we can construct the eigenvalue problem. If we denote the real frequency for the single adiabaticity as ω_{MHD} and that for the double adiabaticity as ω_{DA}, we can obtain the relation

$$\omega_{DA}^{\ 2} = \frac{\omega_{MHD}^{\ 2}\left\langle\frac{\rho|\nabla\psi|^2}{B^2}\right\rangle + C_{DA}}{\left\langle\frac{\rho|\nabla\psi|^2}{B^2}\right\rangle + \frac{\left\langle\rho B^2\right\rangle\left\langle h\kappa_s\right\rangle^2}{\left\langle h^2\right\rangle^2}}. \quad (6)$$

In conclusion, in the highly collisionless plasma, the MHD with double adiabatic laws may be more plausible than MHD with single adiabatic law. In the MHD with double adiabatic laws the interchange instability localized around the rational surface does not occur because of coupling with the slow sound mode. The lack of instability in LHD configuration in inwardly shifted configuration might be explained by this effect.

[1] K. Yamazaki *et al.*, J. Fusion Energy **15**, 7 (1996).
[2] H. Yamada *et al.*, Plasma Phys. Control. Fusion **43**, A55 (2001).
[3] G.L. Chew, M.L. Goldberger, and F.E. Low, Proc. Roy. Soc. Lond. **236A**, 112 (1956).