§2. Long Time Behavior of the Toroidicity-induced Alfvén Eigenmodes

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A large number of theoretical and computational studies have been made on the toroidicity-induced Alfvén eigenmodes (TAE modes). What seems to be lacking, however, is the investigation of their long time behavior. Here, we mean by "long time'' a time which is much longer than the growth time and damping time of TAE modes, and also longer than the slowing-down time of fast ions. Especially, the slowing-down time that is the time to reform the fast-ion distribution function has been neglected in most of the literature.

We have carried out a simulation where the magnetohydrodynamic equations are solved along with the following equation for time evolution of the fast-ion distribution function,

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \mathbf{x}}\mathbf{v}_d f - \frac{\partial}{v^2 \partial v} v^2 a f + \nu \frac{\partial}{v^2 \partial v}[1+(\frac{v^3}{v_{\mathrm{crit}}^3})]f + \frac{S(\mathbf{x})}{v^2}\delta(v-v_0), \tag{1}$$

where v_{crit} and v_0 are the critical velocity and the birth velocity of fast ions, respectively. On the right-hand-side the third term with a coefficient ν and the fourth term represent the slowing down and the source of fast ions, respectively. Since the major part of the energy transfer from fast ions to TAE modes is contributed by passing particles, we consider only the parallel velocity component. We, however, adopt a Jacobian for the three-dimensional velocity space to be consistent with the slowing-down term. The effect of fast ions on the magnetohydrodynamic fluid is considered in the magnetohydrodynamic momentum equation [1-2].

When the first and second terms on the right-hand-side of Eq. (1) are neglected and v_{crit} is much smaller than v_0, steady fast-ion pressure is obtained as

$$p_{\mathrm{f}}(\mathbf{x}) = m_{\mathrm{f}} v_0^2 v_{\mathrm{crit}}^3 S(\mathbf{x})/\nu. \tag{2}$$

We introduce the slowing-down time τ_s, which is defined by

$$\tau_s = p_{\mathrm{f}}(\mathbf{x})/[m_{\mathrm{f}} v_0^2 S(\mathbf{x})]. \tag{3}$$

The spatial profile of $S(x)$ is set to be in proportion to the square of the background pressure. We can specify the system by two parameters τ_s and p_0 which is the fast-ion pressure on the magnetic axis in the steady state described above.

In this article, we report the results of two simulation runs. The initial condition is a magnetohydrodynamic equilibrium where the aspect ratio is 3. The minor radius is 16 times larger than the Larmor radius of a fast ion with the Alfvén velocity. Finite viscosity is considered in the magnetohydrodynamic momentum equation to mimic the damping of TAE modes. The viscosity yields a finite damping time of $130\ \tau_A$ for an $n=2$ TAE mode which is observed in these simulation results.

Fig. 1 shows the time evolution of magnetic field fluctuations with $p_0=4\%$ (normalized by the magnetic pressure) and $\tau_s=100\ \tau_A$. The $n=2,\ 3,\ 4$ TAE modes reach roughly a steady saturation levels after $500\ \tau_A$. On the other hand, the story is quite different for a larger τ_s. Fig. 2 shows the same as Fig. 1 for $p_0=4\%$ and $\tau_s=1000\ \tau_A$. TAE modes with $n=1$-6 show a pulsating behavior. This behavior can be naturally interpreted as a cycle of reformation of fast-ion distribution, growth of TAE modes due to fast-ion drive, global flattening of fast-ion distribution or fast-ion loss due to TAE activity, and damping of TAE modes due to less fast-ion drive. It is interesting to note that all modes behave coherently at the second and third bursts.

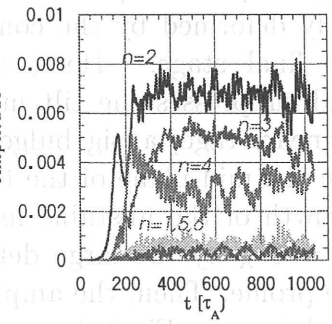

Fig. 1. Time evolution of magnetic field fluctuations for $p_0=4\%$ and $\tau_s=100\ \tau_A$.

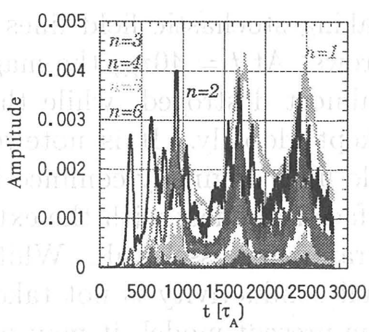

Fig. 2. Time evolution of magnetic field fluctuations for $p_0=4\%$ and $\tau_s=1000\ \tau_A$.

References

[1] Y. Todo, et al., Phys. Plasmas **2**, 2711 (1995).
[2] Y. Todo and T. Sato, Phys. Plasmas **5**, 1321 (1998).