§22. Data Analysis Study for Microwave Tomography

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The purpose of study is to develop the image reconstruction method for the microwave computerized tomography (microwave CT) with high space resolution. A new microwave imaging system is under development on the basis of the advanced measurement technology that has been built up for the microwave imaging reflectometry (MIR) of high temperature plasma.[1,2] The system will serve purposes of practical imaging such as the breast cancer detection and the concrete pillar diagnostics. The system will be able to emit microwaves to objects with frequency variable and higher than those of the conventional practical imaging systems, and receive the scattered waves with phase information.

In view of the breast cancer detection, the electric property of human body is dominated by that of water. The permittivity of water decreases at the frequency higher than 10 GHz while the conductivity reaches a maximum at the range from 20 to 30 GHz. On the other hand, in general, the microwave imaging has higher space resolutions at higher frequencies. With this consideration and with regard to the history of medical imaging research, our choice of the frequency from 8.5 to 14 GHz in system design may be wise. The scattered microwaves will be received by a line array of 16 antennas and detected as complex signals by a quadrature detection circuit. A geometry of imaging test for phantoms is shown in Fig. 1.

In paying attention to the confocal method using the ultrawideband radar technique, which looks promising for the early detection of breast cancer,[3] our study of data analysis has started by reviewing the works of image reconstruction from the classical formulation of diffraction tomography to the recently advanced nonlinear methods.

When an object is illuminated in the two-dimensional scheme by a TM incident wave $e^i(\boldsymbol{r})$ with time dependence of $\exp(-i\omega t)$, the electric fields $e(\boldsymbol{r})$ in the object and $e(\boldsymbol{s})$ at a receiver are expressed as

$$e(\boldsymbol{r}) = e^i(\boldsymbol{r}) + \iint_S k_0^2 c(\boldsymbol{r}')e(\boldsymbol{r}')G(\boldsymbol{r},\boldsymbol{r}')d\boldsymbol{r}' \qquad (1)$$

and

$$e(\boldsymbol{s}) = \iint_S k_0^2 c(\boldsymbol{r}')e(\boldsymbol{r}')G(\boldsymbol{s},\boldsymbol{r}')d\boldsymbol{r}' . \qquad (2)$$

Here, $c(\boldsymbol{r})$ and $G(\boldsymbol{r}, \boldsymbol{r}')$ denote the contrast function of complex permittivity and the Green function, respectively, and k_0 is the wavenumber in free space. When $e(\boldsymbol{s})$ is observed at several points in a multi-transmitter configuration, determining $c(\boldsymbol{r})$ from the acquired data is a nonlinear problem of image reconstruction. Under the Born approximation the problem becomes linear, and one gets a Fourier-slice theorem as a useful mathematical tool when plane incident waves are used.

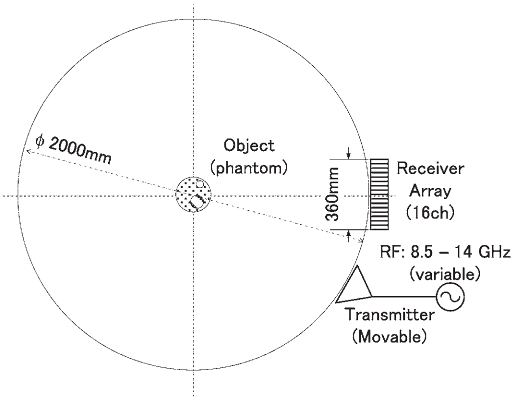

Fig. 1 Geometry of microwave imaging experiment.

In general, however, the relation between the contrast function (image) and the received scattering waves is nonlinear. After pixellating the object region, the problem can be written in an algebraic equation

$$A(\boldsymbol{c})\boldsymbol{c} = \boldsymbol{e}^s \qquad (3)$$

in order to determine the unknown image vector $\boldsymbol{c}$ from the data vector $\boldsymbol{e}^s$ of the whole receiving system. The coefficient $A(\boldsymbol{c})$ is a matrix composed of the matrices $A_l(\boldsymbol{c})=k_0{}^2G_l^sE_l$, where E_l is a diagonal matrix including the vector $\boldsymbol{e}_l = (I - k_0{}^2G^rC)^{-1}\boldsymbol{e}_l^i$ with a $\boldsymbol{c}$-related diagonal matrix C; G^r and G_l^s are matrix expressions of the Green functions; l is the number of transmitter.[4]

This inversion equation is ill-posed with a nonlinearity that should appear strongly whenever the objects are highly contrasted and whenever the decay of the incident microwave due to the conductivity is large. For well-regularized solutions and consequently for high space resolutions, iterative procedures of Newton type and conjugate gradient type are being investigated in taking into account the features of our microwave system.

Owing to our optical technique for plane wave transmission in MIR, a related linear solution of Eq. (3) based on the Born approximation may be useful for the initial solution in iteration. Also, the feature of variable frequency may serve improving the space resolution in image reconstruction by utilizing the multi-frequency information. Surely, the finite difference time domain (FDTD) analysis of microwave field well-developed in nuclear fusion research should be useful for the forward calculation of the coefficient matrix $A(\boldsymbol{c})$. Our experimental study using phantoms starts under the condition which is suitable for explaining the basic characteristics of microwave imaging.

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2) Y. Nagayama, S. Yamaguchi, Z. Shi, Y. Kogi, A. Mase et al.: Plasma and Fusion Res., **3** (2008) 053.
3) S.C. Hagness, A. Taflove, and J.E. Bridges: IEEE Trans. Biomed. Eng., **BE-45** (1998) 1470.
4) A. Franchois and C. Pichot: IEEE Trans. Antennas Propagat., **AP-45** (1997) 203.