§13. Reconstruction of Soft X-Ray Emissivity Profile in CHS by Means of Singular-Value Decomposition Technique

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The soft X-ray (SX) signals measured by the SX detector array are obtained as signals integrated along the line of sight of the detector. Therefore, a local SX emissivity must be derived from a set of these line-integrated signals if detailed internal structure of MHD instabilities is required. The measured SX signals are expressed by linear algebraic equations under an assumption that SX emissivity is constant on each magnetic surface

$$S_i = \sum_j A_{ij} P_j \qquad (1)$$

or matrix expression

$$\mathbf{S} = \mathbf{AP} \qquad (2)$$

where S_i is the i-th channel measured SX signal (i = 1, 2, ..., M), A_{ij} is path length of j-th magnetic surface, which is sliced off by the i-th line of sight, and P_j is the local SX emissivity on the j-th magnetic surface (j = 1, 2, ..., N). This equation can be solved by a so-called direct method, for example, Gaussian elimination method or LU decomposition. But, the direct methods often fail to give satisfactory results if **A** is close to a singular matrix.

In order to overcome this problem, we can use the singular-value decomposition (SVD). SVD is able to deal with matrices that are singular or near-singular, and will give the best approximate solution of eq. (2) in the least-square sense. SVD transforms any $M \times N$ matrix **A** to the product of following three matrices

$$\mathbf{A} = \mathbf{UWV}^{T} \qquad (3)$$

where **U** is an $M \times N$ orthogonal matrix, **W** is an $N \times N$ diagonal matrix ($W_{ij} = \delta_{ij} w_j$) and **V** is an $N \times N$ orthogonal matrix. The quantities w_j are called singular-value of **A**. The solution that minimizes the residual error is given by following form

$$\mathbf{P} = \mathbf{V}\{\mathrm{diag}(1/w_j)\}(\mathbf{U}^{T}\mathbf{S}) \qquad (4)$$

Matrix **A** is determined by the geometical relationship between lines of sight of SX detectors and magnetic surface positions. Here, we use the equilibrium data of the magnetic surfaces of the CHS plasmas in the averaged plasma beta (β = 0, 0.2, 0.5 and 0.8 %), which are obtained by the VMEC-code. For a simple calculation of **A**, we adopt an assumption that the shape of cross-section of the magnetic surface remains unchanged in low-β plasmas with the increase in β up to 1 %. On this assumption, the magnetic surfaces of the CHS plasma can be represented by the magnetic axis position R_{ax}. We made data sets of the magnetic surfaces from β = 0 % to β = 0.8 % where the shift of each magnetic surface for various β is approximated by a polynominal fit. Next, we discuss how to determine the optimum number of layers N to be used in the calculation. We use the so-called Akaike Information Criterion (AIC) to determine the adequate number of layers. This is defined as follows,

$$\mathrm{AIC} = M \log R + 2N \qquad (5)$$

where R is the squared sum of the residual error. This criterion results from a trade-off between the likelihood of a model and a penalty for making the model too complex.

We check the validity of this criterion using test SX profile. The test SX profile is provided by line integration of assumed local SX emissivity profile along the line of sight, where the number of magnetic surface with constant SX emissivity N (= 10) and R_{ax} (= 0.96 m) is fixed. The contour of AIC in the plane of R_{ax} and N is shown in Fig. 1. The minimum-AIC is obtained for R_{ax} = 0.9585 m and N = 10, of which values agree well with the assumed ones R_{ax} = 0.96 m and N = 10. It is concluded from the above test analysis that the concept of AIC can be effectively applied to analysis of SX data.

We actually apply this reconstruction method to the SX data taken in a plasma with sawtooth activities on CHS. Typical sawtooth oscillations observed in CHS are shown in Fig. 2(a). Figure 2(b) shows SX emissivity profiles just before (A) and after (B) the crash, which are obtained by the above-mentioned numerical method based on SVD technique, where the precursor and postcursor oscillations are numerically filtered. The SX emissivity profile after the crash clearly exhibits the change only in an annular region around ρ ~ 0.5. This behavior is similar to the off-axis or annular crash observed in the reversed shear tokamak plasmas.

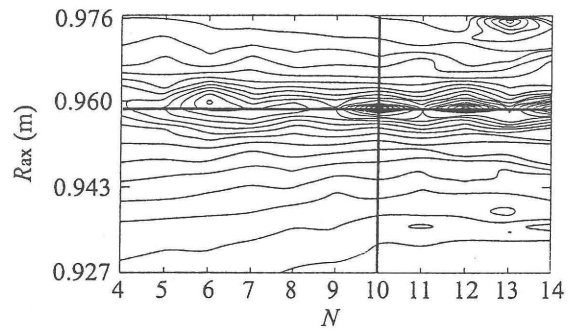

Fig. 1 The contour map of AIC in the plane of R_{ax} and N.

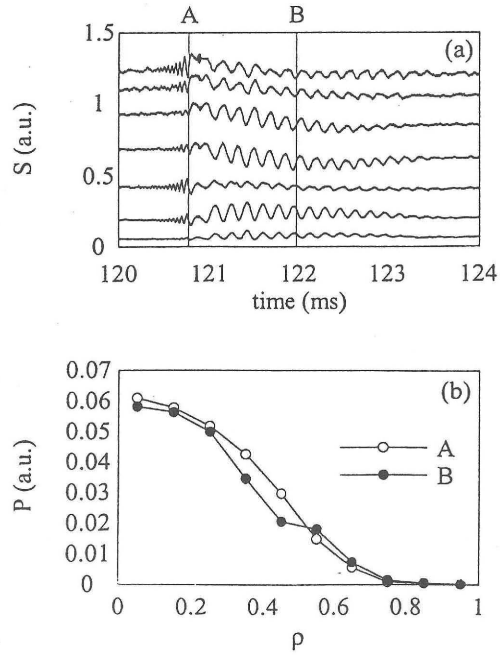

Fig. (2) (a) Typical time evolution of SX signals for one period of a sawtooth oscillation. (b) Reconstructed SX emissivity profile just before (A) and after (B) the sawtooth crash.