§28. Application of Collisionless Kinetic-Fluid Closure to the Three-Mode Ion Temperature Gradient Driven System

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The new collisionless kinetic-fluid closure model is applied to the three-mode ion temperature gradient (ITG) driven system [1]. We consider a rectangular domain of $L_x \times L_y$ in the x-y plane with a uniform external magnetic field $\mathbf{B} = B_0(\hat{\mathbf{z}} + \theta\hat{\mathbf{y}})$ ($|\theta| \ll 1$), where $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ denote the unit vectors in the y- and z-directions, respectively. The system is assumed to be homogeneous in the z-direction ($\partial/\partial z = 0$). The background density and temperature gradients are assumed to exist in the x-direction, and their gradient scale lengths are given by $L_n = -(d\ln n_0/dx)^{-1} (> 0)$ and $L_T = -(d\ln T_i/dx)^{-1} (> 0)$, respectively. We employ the periodic boundary conditions in both x and y directions. In the three-mode ITG system, the perturbation part of the ion distribution function is given by $f = 2\mathrm{Re}[f_1(v_\parallel, t)\cos(2\pi x/L_x)\exp(2\pi i y/L_y)] - 2h(v_\parallel, t)\sin(4\pi x/L_x)$, and the governing equations for f_1, h, and the electostatic potential $\phi = 2\mathrm{Re}[\phi_1(t)\cos(2\pi x/L_x)\exp(2\pi i y/L_y)]$ are given by

$$(\partial_t + ik\Theta v_\parallel)f_1(v_\parallel, t) + 2ik^2\phi_1(t)h(v_\parallel, t) = -ik\phi_1(t)G(v_\parallel), \quad (1)$$

$$\partial_t h(v_\parallel, t) = 4k^2\mathrm{Im}[\phi_1^*(t)f_1(v_\parallel, t)], \quad (2)$$

$$\phi_1(t) = \int dv_\parallel \, f_1(v_\parallel, t), \quad (3)$$

where $L_x = L_y = 1/k$ and $T_i = T_e$ (T_e: the electron temperature) are assumed and $G(v_\parallel) \equiv [1 + (v_\parallel^2 - 1)\eta_i/2 + \Theta v_\parallel]e^{-v_\parallel^2/2}/(2\pi)^{-1/2}$. Here, we have used dimensionless normalized variables $x = x'/\rho_i$, $y = y'/\rho_i$, $v = v'/v_t$, $t = t'v_t/L_n$, $f = f'L_n v_t/\rho_i n_0$, and $\phi = e\phi' L_n/T_i\rho_i$, where prime represents a dimensional quantity, $v_t = \sqrt{T_i/m_i}$ is the ion thermal velocity, $\rho_i = v_t/\Omega_i$ is the ion thermal gyroradius, and $\Omega_i = eB/m_i c$ is the ion gyrofrequency. Two important parameters Θ and η_i are given by $\Theta = \theta L_n/\rho_i$ and $\eta_i = L_n/L_T$, respectively.

Taking the velocity moments of Eqs.(1) and (2), we obtain

$$\partial_t n_1 + ik(\Theta u_1 + \phi_1) = 0, \quad (4)$$

$$\partial_t u_1 + ik\Theta(n_1 + T_1 + \phi_1) + 2ik^2\phi_1 \, u_h = 0, \quad (5)$$

$$\partial_t u_h - 4k^2\mathrm{Im}(\phi_1^* u_1) = 0, \quad (6)$$

$$\partial_t T_1 + ik[\Theta(2u_1 + q_1) + \eta_i\phi_1] + 2ik^2\phi_1 \, T_h = 0, \quad (7)$$

$$\partial_t T_h - 4k^2\mathrm{Im}(\phi_1^* T_1) = 0, \quad (8)$$

where $[n_1(t), u_1(t), T_1(t), q_1(t)] = \int_{-\infty}^{\infty} dv_\parallel \, f_1(v_\parallel, t)[1, v_\parallel, (v_\parallel^2 - 1), (v_\parallel^3 - 3v_\parallel)]$ and $[u_h(t), T_h(t)] = \int_{-\infty}^{\infty} dv_\parallel \, h(v_\parallel, t)[v_\parallel, (v_\parallel^2 - 1)]$. We also obtain from Eq.(3),

$$n_1 = \phi_1. \quad (9)$$

In our nondissipative closure model (NCM), we have

$$q_1 = C_{T1}T_1 + C_{u1}u_1, \quad (10)$$

where C_{T1} and C_{u1} are real coefficients determined so as to give a valid relation for the linear normal mode solution and its complex-conjugate solution. On the other hand, in the Hammett-Perkins model, $q_1 = -2(2/\pi)^{1/2}ik\Theta T_1$ is used. Figure 1 shows $|\phi_1(t)|$ obtained by numerically solving these fluid equations (4)–(10). The exact nonlinear solution of the kinetic equations (1)–(3) discovered by Watanabe, Sugama, and Sato (WSS) [2] is reproduced as a solution of Eqs.(4)–(10) since the NCM in Eq.(10) is completely satisfied by the WSS solution. The case of the most simple closure, in which $q_1 = 0$, and that of the Hammett-Perkins closure $q_1 = -2(2/\pi)^{1/2}ik\Theta T_1$ are plotted for comparison to the WSS solution in Fig.1. In the Hammett-Perkins model, the potential $|\phi_1(t)|$ is saturated at a certain amplitude, which is in contrast to the periodic amplitude oscillation shown by the NCM and the $q_1 = 0$ model.

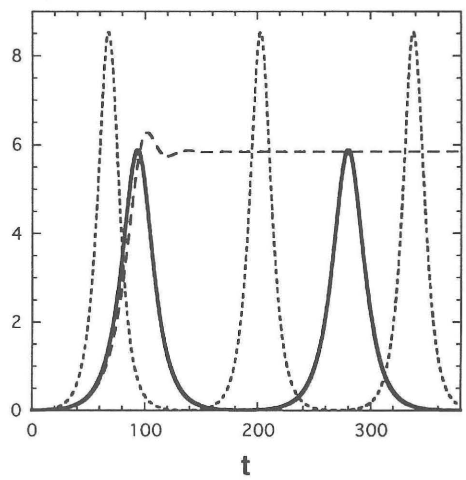

Fig.1. Numerical solutions of the fluid system of Eqs.(4)–(9) for $k = 0.1$, $\Theta = 1$, and $\eta_i = 10$. A solid curve represents $|\phi_1(t)|$ obtained by using the NCM in Eq.(10). Results obtained by using the Hammett-Perkins model and the $q_1 = 0$ model are also shown by dashed and dotted curves, respectively.

References
1) Sugama, H., Watanabe, T.-H., and Horton, W., Phys. Plasmas **8** (2001) 2617.
2) Watanabe, T.-H., Sugama, H., and Sato, T., Phys. Plasmas **7** (2000) 984.