§ 10. Radial Electric Field Dependence of Neoclassical Poloidal and Toroidal Viscosity Coefficients

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We have presented a novel method to obtain the full neoclassical transport matrix for general toroidal plasmas by using the solution of the linearized drift kinetic equation with the pitch-angle-scattering collision operator [1]. This method can also be applied to investigation of the $\mathbf{E}\times\mathbf{B}$ drift effects on the neoclassical transport coefficients based on the drift kinetic equation given by

$$Vf_{a1} - C_a^L(f_{a1}) = -\mathbf{v}_{da}\cdot\nabla f_{aM} + \frac{e_a}{T_a}v_{\parallel}B\frac{\langle BE_{\parallel}\rangle}{\langle B^2\rangle}f_{aM},$$

where the operator $V \equiv V_{\parallel} + V_E$ consists of the parallel motion part

$$V_{\parallel} = v\xi\mathbf{b}\cdot\nabla - \frac{1}{2}v(1-\xi^2)(\mathbf{b}\cdot\nabla\ln B)\frac{\partial}{\partial\xi},$$

and the $\mathbf{E}\times\mathbf{B}$ drift part

$$V_E \equiv \mathbf{v}_E\cdot\nabla \equiv \frac{cE_s}{\langle B^2\rangle}\nabla s\times\mathbf{B}\cdot\nabla,$$

with ∇ taken for $(v, \xi \equiv v_{\parallel}/v)$ being fixed. The $\mathbf{E}\times\mathbf{B}$ drift operator V_E has the same form as employed in the DKES [2]. Here, we assume the incompressibility conditions $\nabla\cdot\mathbf{u}_a = \nabla\cdot\mathbf{q}_a = 0$ and the stellarator symmetry $B(s,\theta,\zeta) = B(s,-\theta,-\zeta)$.

In helical systems, the polidal and toroidal viscosities of $[\langle\mathbf{B}_P\cdot(\nabla\cdot\boldsymbol{\pi}_a)\rangle, \langle\mathbf{B}_P\cdot(\nabla\cdot\boldsymbol{\Theta}_a)\rangle, \langle\mathbf{B}_T\cdot(\nabla\cdot\boldsymbol{\pi}_a)\rangle, \langle\mathbf{B}_T\cdot(\nabla\cdot\boldsymbol{\Theta}_a)\rangle]$ are related to the polidal and toroidal flows $[\langle u_a^{\theta}\rangle/\chi', \frac{2}{5p_a}\langle q_a^{\theta}\rangle/\chi', \langle u_a^{\zeta}\rangle/\psi', \frac{2}{5p_a}\langle q_a^{\zeta}\rangle/\psi',]$ by

$$\begin{bmatrix}\langle\mathbf{B}_P\cdot(\nabla\cdot\boldsymbol{\pi}_a)\rangle\\ \langle\mathbf{B}_P\cdot(\nabla\cdot\boldsymbol{\Theta}_a)\rangle\\ \langle\mathbf{B}_T\cdot(\nabla\cdot\boldsymbol{\pi}_a)\rangle\\ \langle\mathbf{B}_T\cdot(\nabla\cdot\boldsymbol{\Theta}_a)\rangle\end{bmatrix} = \begin{bmatrix} M_{a1PP} & M_{a2PP} & M_{a1PT} & M_{a2PT}\\ M_{a2PP} & M_{a3PP} & M_{a2PT} & M_{a3PT}\\ M_{a1PT} & M_{a2PT} & M_{a1TT} & M_{a2TT}\\ M_{a2PT} & M_{a3PT} & M_{a2TT} & M_{a3TT}\end{bmatrix}\begin{bmatrix}\langle u_a^{\theta}\rangle/\chi'\\ \frac{2}{5p_a}\langle q_a^{\theta}\rangle/\chi'\\ \langle u_a^{\zeta}\rangle/\psi'\\ \frac{2}{5p_a}\langle q_a^{\zeta}\rangle/\psi'\end{bmatrix}.$$

Here, the Onsager-symmetric poloidal and toroidal viscosity coefficients M_{ajPP}, M_{ajPT}, and M_{ajTP} $(j = 1,2,3)$ are also written in the form of the energy integral:

$$[M_{ajPP}, M_{ajPT}, M_{ajTT}] = n_a\frac{2}{\sqrt{\pi}}\int_0^{\infty}dK\sqrt{K}e^{-K} \times\left(K-\frac{5}{2}\right)^{j-1}[M_{aPP}(K), M_{aPT}(K), M_{aTT}(K)],$$

where $M_{aPP}(K)$, $M_{aPT}(K)$, and $M_{TT}(K)$ represent contributions of monoenergetic particles to M_{ajPP}, M_{ajPT}, and M_{ajTT}, respectively.

Figure 1 shows the normalized monoenergetic neoclassical viscosity coefficients $[M_{PP}^*, M_{PT}^*, M_{TT}^*] \equiv [M_{PP}(K), M_{PT}(K), M_{TT}(K)]/[(4\pi^2/V')mv_T(\psi'\chi')^2K^{3/2}]$ as a function of cE_s/v, which are obtained by combining our method with numerical output of the DKES for the magngetic field strength given by $B = B_0[1 - \epsilon_t\cos\theta_B - \epsilon_h\cos(l\theta_B - n\zeta_B)]$ with $B_0 = 1\ T$, $\epsilon_t = 0.1$, $\epsilon_h = 0.05$, $l = 2$, and $n = 10$. Here, $\nu_D/v = 3\times10^{-6}$ is used, which corresponds to the $1/\nu$ regime for the case of $E_s = 0$. In Fig. 1, $M_{PP} \simeq -M_{PT} \simeq M_{TT}$ (which implies small parallel viscosities) and their reduction with increasing cE_s/v are clearly seen. The E_s-dependent neoclassical transport coefficients for radial fluxes and parallel currents can be calculated as well.

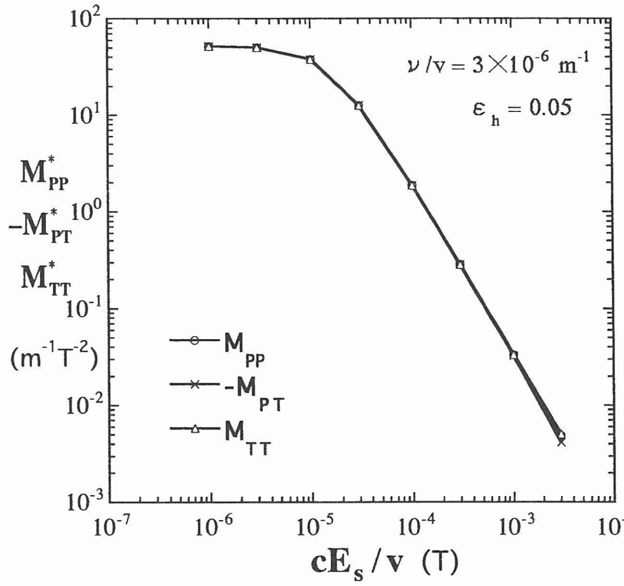

Fig.1. Poloidal and toroidal viscosity coefficients as a function of cE_s/v for $\epsilon_t = 0.1$, $\epsilon_h = 0.05$ and $\nu_D/v = 3\times10^{-6}$.

References

1) Sugama, H., and Nishimura, S., Phys. Plasmas **9** (2002) 4637.

2) van Rij, W. I. and Hirshman, S. P., Phys. Fluids B **1** (1989) 563.