§20. On the Saturation of Multihelicity Resistive Interchange Modes

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Nonlinear amplitude equations of the multi-helicity modes near marginally stable states are given by [1,2]

$$dA_n/dt = \gamma A_n - \sum_{n_1,n_2} V_{n-n_1,n-n_2} A_{n-n_1+n_2} A_{n_1} A_{n_2}$$

where $A_n(t)$ ($n = 0, \pm 1, \pm 2, \cdots$) denote the amplitudes of the multi-helicity modes with the lowest poloidal mode number $m = 1$ and the toroidal mode number n for the time t. The potential fluctuation is approximately given in terms of the linear eigenfunctions ϕ_1 for the dominant $m = 1$ modes as $\tilde{\phi} = \sum_{n=-\infty}^{\infty} A_n \phi_1(x + n\Delta) \sin[2\pi(y/L_y + nz/L_z)]$ where x, y, and z are the coordinates in the radial, poloidal, and toroidal directions. Here $m = 1$ and $n = 1$ corresponds to the largest poloidal and toroidal wavelengths not in the whole toroidal region but in the local slab system, which are denoted by L_y and L_z. The radial interval between the adjacent $m = 1$ mode rational surfaces is given by $\Delta \equiv L_s L_y / L_z$ (L_s: the magnetic shear length) while all the $m = 1$ modes have the same linear growth rate $\gamma(> 0)$. The coefficients V_{mn} of the nonlinear term is calculated from the linear eigenfunctions. The coefficient V_{00} represents the intensity of the self-interaction or the nonlinear coupling of the modes with the same helicity and V_{mn} for $(m, n) \neq (0, 0)$ represents that of the interaction between the modes with different helicities.

If we use $\tau = \gamma t$, $X_n = (V_{00}/\gamma) A_n^2$, $\lambda = (V_{01} + V_{11})/V_{00}$ and retain only the self-interaction and interaction between adjacent modes, we obtain

$$\frac{1}{2}\frac{dX_n}{d\tau} = X_n(1 - X_n - \lambda X_{n-1} - \lambda X_{n+1}).$$

Here the parameter λ measures the strength of the nonlinear interaction between the adjacent $m = 1$ modes and is an increasing function of the ratio of the radial width W of the $m = 1$ mode structure $\phi_1(x)$ to the interval Δ. We easily obtain the following nontrivial stationary solutions of the above equations:

$$\text{(I)} \qquad X_n = 1/(1 + 2\lambda) \quad (\text{for all } n)$$

$$\text{(II)} \qquad X_n = \begin{cases} 0 & (\text{for even } n) \\ 1 & (\text{for odd } n) \end{cases}$$

Linear analysis shows that, for the case (a) $\lambda < 1/2$, the solutions (I) and (II) are stable and unstable, respectively, while, for the case (b) $\lambda > 1/2$, the solutions (I) and (II) are unstable and stable, respectively. Radial profiles of the mode structures corresponding to these solutions are schematically shown in Figs.1 (a) and (b). Thus, when the multihelicity interaction becomes strong enough, one of the two adjacent mode amplitudes vanishes. This tendency is also confirmed by the nonlinear simulation of the multihelicity resistive interchange modes [2].

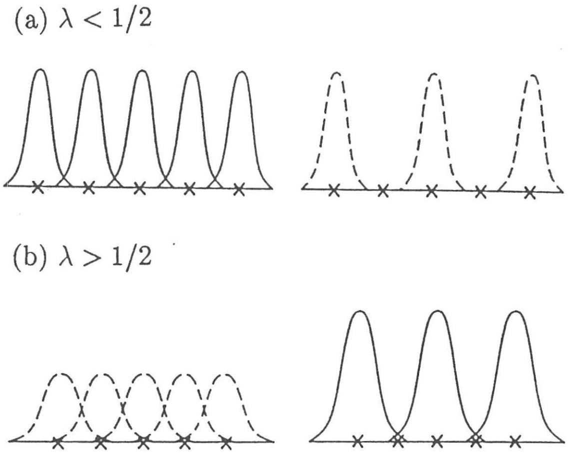

Fig. 1. Radial profiles of the mode structures corresponding to the solutions (I) (left) and (II) (right) for the cases (a) $\lambda < 1/2$ and (b) $\lambda > 1/2$. Solid (dashed) curves correspond to stable (unstable) solutions.

References
1) Sugama, H., Nakajima, N., and Wakatani, M. : Phys. Fluids B 3 (1991) 3290.
2) Sugama, H. and Horton, W. : Comments Plasma Phys. Controlled Fusion 17 (1996) 277.