## §15. Dependence of Neoclassical Viscosity and Inertia Force on Radial Electric Field

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Effects of radial electric fields on the neoclassical viscosity and the inertia force in axisymmetric toroidal plasmas are investigated [1]. We use the drift kinetic equation for the perturbed distribution function $f_1(\mathbf{x}, v, \xi)$ ($\xi \equiv v_\parallel / v \equiv \cos\alpha$: the cosine of the pitch angle) given by

$$\left(v\xi \mathbf{b}\cdot\nabla - \frac{1}{2}v(1-\xi^2)\mathbf{b}\cdot(\nabla \ln B)\frac{\partial}{\partial \xi} + \mathcal{L}_E - C\right) f_1$$
$$= -\frac{m_i c}{2eB}v^2(1+\xi^2)\mathbf{b}\times(\nabla \ln B)\cdot\nabla r \frac{\partial f_M}{\partial r} \qquad (1)$$

where $C$ denotes the linearized collision operator, and the effect of the radial electric field $E_r$ is represented by $\mathcal{L}_E = \mathbf{v}_E \cdot \nabla + \dot{v}_E \frac{\partial}{\partial v} + \dot{\xi}_E \frac{\partial}{\partial \xi}$, with $\mathbf{v}_E \equiv (cE_r/B)\nabla r \times \mathbf{b}$, $\dot{v}_E = \frac{1}{2}(cE_r/B)v(1+\xi^2)(\nabla\times\mathbf{b})\cdot\nabla r$, and $\dot{\xi}_E = \frac{1}{2}(cE_r/B)v\xi(1-\xi^2)(\nabla\times\mathbf{b})\cdot\nabla r$. Here, effects of the nonlinear inertia term $\mathbf{v}_E\cdot\nabla\mathbf{v}_E$ are not considered. The parallel momentum balance equation for the axisymmetric case are derived from the drift kinetic equation (1) as

$$\langle \mathbf{B}\cdot(\nabla\cdot\boldsymbol{\pi})\rangle_{\rm plateau} - 2m_i n_i \langle u_\parallel \mathbf{v}_E\cdot\nabla B\rangle = \frac{e\Psi'\langle B^2\rangle}{cI}\,\Gamma_{\rm orbit} \qquad (2)$$

where $\langle\cdots\rangle$ denotes the flux-surface average. In Eq.(2), $\langle \mathbf{B}\cdot(\nabla\cdot\boldsymbol{\pi})\rangle_{\rm plateau}$ represents the contribution of ions in the plateau region $[v < (r/R)^{-3/2}Rq\nu_{ii}]$ to the surface-averaged parallel ion viscosity [the Pfirsch-Schlüter region ($v < Rq\nu_{ii}$) is assumed to be negligibly small]. The right-hand side of Eq.(2) represents a driving force for the poloidal rotation due to the orbit loss $\Gamma_{\rm orbit}[= -(cI/e\Psi'\langle B^2\rangle)\langle \mathbf{B}\cdot(\nabla\cdot\boldsymbol{\pi})\rangle_{\rm banana}]$ of ions in the banana region $[v > (r/R)^{-3/2}Rq\nu_{ii}]$. The inertia force $-2m_i n_i\langle u_\parallel \mathbf{v}_E\cdot\nabla B\rangle = m_i\langle \mathbf{B}\cdot[\nabla\cdot\{n_i u_\parallel(\mathbf{b}\mathbf{v}_E + \mathbf{v}_E\mathbf{b})\}]\rangle$ results from the $\mathcal{L}_E$ term in Eq.(1).

The approximate analytical solution of Eq.(1) for the plateau regime is obtained, which gives the plateau parallel viscosity and the inertia force as

$$\langle \mathbf{B}\cdot(\nabla\cdot\boldsymbol{\pi})\rangle_{\rm plateau} = \frac{\sqrt{\pi}}{4}n_i m_i \frac{v_{Ti}}{Rq}B_0^2\left(\hat{\mu}_{\pi 1}\bar{u}_\theta + \hat{\mu}_{\pi 2}\frac{2\bar{q}_\theta}{5p_i}\right)$$

$$-2m_i n_i\langle u_\parallel \mathbf{v}_E\cdot\nabla B\rangle = \frac{\sqrt{\pi}}{4}n_i m_i \frac{v_{Ti}}{Rq}B_0^2\left(\hat{\mu}_{E1}\bar{u}_\theta + \hat{\mu}_{E2}\frac{2\bar{q}_\theta}{5p_i}\right) \qquad (3)$$

with

$$\begin{bmatrix}\hat{\mu}_{\pi j}\\ \hat{\mu}_{Ej}\end{bmatrix} = 8\sum_{m\geq 1} m^2(\epsilon_m^2 + \delta_m^2)\int_0^{\nu_{*i}} dx e^{-x^2}x^5$$
$$\times \int_{-1}^{1} d\xi \Delta(m\{\xi - M_p(v)\})\{P_2(\xi) - 2M_p(v)\xi\}$$
$$\times \left(x^2 - \frac{5}{2}\right)^{j-1}\begin{bmatrix}P_2(\xi)\\ -2M_p(v)\xi\end{bmatrix} \qquad (4)$$

where $\Delta(m\{\xi - M_p(v)\}) \equiv \pi^{-1}\bar{\nu}/[(m\{\xi - M_p(v)\})^2 + \bar{\nu}^2]$, $M_p(v) \equiv cIE_r/(\Psi' B_0 v)$, and $B = B_0[1 - \sum_{m\geq 1}\{\epsilon_m\cos(m\theta) + \delta_m \sin(m\theta)\}]$ are used. These analytical results are in good agreement with the numerical solutions by the DKES code as shown in Fig.1.

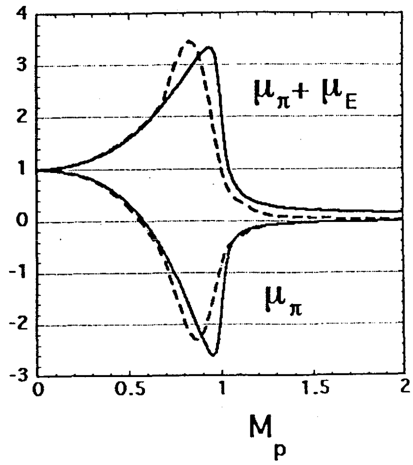


**Fig.1.** Coefficients $\hat{\mu}_\pi$ and $\hat{\mu}_\pi + \hat{\mu}_E$ as a function of $M_p$. They are normalized by the value of $\hat{\mu}_\pi$ for $M_p = 0$. Solid and dashed curves correspond to analytical and numerical results, respectively.

References

1) Sugama, H., Furukawa, M., Wakatani, M., and Horton, W. : Proceedings of the Joint Varenna-Lausanne International Workshop on Theory of Fusion Plasmas (1999) p.455.