§17. Effects of Electrostatic Fluctuations on Neoclassical Transport

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We start from an ensemble-averaged kinetic equation :

$$\frac{\partial f_a}{\partial t}+\boldsymbol{v}\cdot\nabla f_a+\frac{e_a}{m_a}\left(\boldsymbol{E}+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right)\cdot\frac{\partial f_a}{\partial \boldsymbol{v}}=C_a+\mathcal{D}_a$$

where C_a is a collision term and $\mathcal{D}_a$ is fluctuation-averaged term defined by

$$\mathcal{D}_a=-\frac{e_a}{m_a}\left\langle\hat{\boldsymbol{E}}\cdot\frac{\partial \hat{f}_a}{\partial \boldsymbol{v}}\right\rangle_{ens}, \quad \hat{\boldsymbol{E}}=-\nabla\hat{\phi}.$$

Here the distribution function (the electric field) is devivided into the average part f_a ($\boldsymbol{E}$) and the fluctuating part $\hat{f}_a$ ($\hat{\boldsymbol{E}}$). The forces resulting from the fluctuation term are given by

$$\boldsymbol{K}_{a1}=\int d^3v\mathcal{D}_a m_a\boldsymbol{v}=e_a\langle\hat{n}_a\hat{\boldsymbol{E}}\rangle_{ens}$$

$$\boldsymbol{K}_{a2}=\int d^3v\mathcal{D}_a m_a\boldsymbol{v}\left(\frac{m_a v^2}{T_a}-\frac{5}{2}\right)$$

$$=\frac{e_a}{T_a}\left\langle\frac{5}{2}(\hat{p}_a-\hat{n}_aT_a)\hat{\boldsymbol{E}}+\hat{\pi}_a\cdot\hat{\boldsymbol{E}}\right\rangle_{ens}$$

where we defined the fluctuating density and pressure (scalar and stress parts) as

$$\hat{n}_a=\int d^3v\hat{f}_a, \quad \hat{p}_a=\frac{1}{3}\int d^3v\hat{f}_a m_a v^2$$

$$\hat{\pi}_a=\frac{1}{3}\int d^3v\hat{f}_a m_a\left(\boldsymbol{vv}-\frac{1}{3}v^2\mathsf{I}\right).$$

Consider axisymmetric systems such as tokamaks, for which the magnetic field is given by

$$\boldsymbol{B}=I\nabla\zeta+\frac{1}{2\pi}\nabla\zeta\times\nabla\chi.$$

The anomalous transport induced by the perpendicular fluctuation forces are given by

$$\langle\boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle^{anom} = \left\langle\frac{\nabla\psi}{m_a\Omega_a}\cdot(\boldsymbol{K}_{a1}\times\boldsymbol{n})\right\rangle$$

$$T_a^{-1}\langle\boldsymbol{q}_a\cdot\nabla\psi\rangle^{anom} = \left\langle\frac{\nabla\psi}{m_a\Omega_a}\cdot(\boldsymbol{K}_{a2}\times\boldsymbol{n})\right\rangle.$$

The Pfirsch-Schlüter fluxes induced by the parallel fluctuation forces are given by

$$\langle\boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle_{PS}^{anom}=-\frac{2\pi}{\chi'}\left\langle\frac{\boldsymbol{n}\cdot\boldsymbol{K}_{a1}}{m_a\Omega_a}\left(I-\langle I\rangle\frac{B^2}{\langle B^2\rangle}\right)\right\rangle$$

$$T_a^{-1}\langle\boldsymbol{q}_a\cdot\nabla\psi\rangle_{PS}^{anom}=-\frac{2\pi}{\chi'}\left\langle\frac{\boldsymbol{n}\cdot\boldsymbol{K}_{a2}}{m_a\Omega_a}\left(I-\langle I\rangle\frac{B^2}{\langle B^2\rangle}\right)\right\rangle.$$

The banana-plateau transport equations including the effects of the fluctuations are written as

$$\begin{bmatrix}\langle\boldsymbol{\Gamma}_e\cdot\nabla\psi\rangle_{bp}\\ \langle\boldsymbol{q}_e\cdot\nabla\psi\rangle_{bp}\\ \langle\boldsymbol{q}_i\cdot\nabla\psi\rangle_{bp}\\ \langle B^2\rangle^{-1/2}\langle Bj_{\parallel}\rangle^{(m)}\end{bmatrix}=\mathsf{L}\begin{bmatrix}-n_e^{-1}(dP/d\psi)^{(m)}\\ -T_e^{-1}(dT_e/d\psi)^{(m)}\\ -T_i^{-1}(dT_i/d\psi)^{(m)}\\ \langle B^2\rangle^{-1/2}\langle BE_{\parallel}^{(A)}\rangle^{(m)}\end{bmatrix}$$

where the fluxes and forces are defined by

$$\langle\boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle_{bp}=-\frac{2\pi I}{\chi'}\frac{c\langle\boldsymbol{B}\cdot\nabla\cdot\pi_a\rangle}{e_a\langle B^2\rangle}$$

$$T_a^{-1}\langle\boldsymbol{q}_a\cdot\nabla\psi\rangle_{bp}=-\frac{2\pi I}{\chi'}\frac{c\langle\boldsymbol{B}\cdot\nabla\cdot\Theta_a\rangle}{e_a\langle B^2\rangle}$$

$$\langle Bj_{\parallel}\rangle^{(m)}=\langle Bj_{\parallel}\rangle-\frac{n_e e^2\tau_e}{m_e}\tilde{\sigma}_{\parallel}\langle BE_{\parallel}\rangle^{(m)}-\sqrt{\frac{2}{5}}\tilde{\alpha}_{\parallel}\frac{e\tau_e}{m_e}\langle\boldsymbol{B}\cdot\boldsymbol{K}_{e2}\rangle$$

$$\left(\frac{dP}{d\psi}\right)^{(m)}=\frac{dP}{d\psi}+\frac{\chi'}{2\pi I}\frac{e}{c}\left(\sqrt{\frac{2}{5}}\tilde{\alpha}_{\parallel}\frac{\tau_e}{m_e}\langle\boldsymbol{B}\cdot\boldsymbol{K}_{e2}\rangle+n_e\langle B^2\rangle(W_{e1}-W_{i1})\right)$$

$$\left(\frac{dT_a}{d\psi}\right)^{(m)}=\frac{dT_a}{d\psi}+\frac{\chi'}{2\pi I}\frac{e_a}{c}\left(\frac{2}{5}\tilde{\kappa}_{\parallel}^a\frac{\tau_a}{n_a m_a}\langle\boldsymbol{B}\cdot\boldsymbol{K}_{a2}\rangle-\langle B^2\rangle W_{a2}\right)$$

$$\langle BE_{\parallel}^{(A)}\rangle^{(m)}=\langle BE_{\parallel}^{(A)}\rangle-(n_e e)^{-1}\langle\boldsymbol{B}\cdot\boldsymbol{K}_{e1}\rangle.$$

Here W_{aj} ($a=e,i; j=1,2$) repersents the corrections of the relations between the parallel viscosities and the poloidal flows due to $\mathcal{D}_a$. The transport maxtrix is the same as the standard neoclassical one and is given by

$$\mathsf{L}=\frac{n_e\rho_e^2}{2T_e\tau_e}\left(\frac{2\pi I}{\chi'}\right)^2\varphi\,\mathsf{D}\begin{bmatrix}l_{11}^{ee} & l_{13}^{ee} & l_{13}^{ei} & l_{1E}^{e}\\ l_{31}^{ee} & l_{33}^{ee} & l_{33}^{ei} & l_{3E}^{e}\\ l_{31}^{ie} & l_{33}^{ie} & Al_{33}^{ii} & l_{3E}^{i}\\ l_{E1}^{e} & l_{E3}^{e} & l_{E3}^{i} & l_{EE}\end{bmatrix}\mathsf{D}$$

$$\mathsf{D}=\mathrm{diag}\left(1,\sqrt{\frac{5}{2}}T_e,\sqrt{\frac{5}{2}\frac{T_i}{Z}},-e\frac{\chi'}{2\pi I}|\Omega_e|\tau_e\right)$$

where the dimensionless coefficients φ, A, $l_{11}^{ee},\cdots$ are given in Ref.[1].

References

[1] R. Balescu : *Transport Processes in Plasmas* (North-Holland, Amsterdam, 1988), Vol. 2.