§13. Gryokinetic Field Theory

Sugama, H.

The gyrokinetic theory is a basic framework to describe microinstabilities, turbulence, and resultant anomalous transport observed in magnetically confined plasmas. Basic equations for the gyrokinetic theory are the gyrokinetic equations for the particle distribution functions and the Poisson-Ampère equations for the electromagnetic fields. A modern derivation of the gyrokinetic equation is based on the Hamiltonian and Lagrangian formulations. The resultant gyrokinetic equation describes the total distribution function as an invariant along the particle motion. There, the motion equation is derived from the gyrophase-independent Hamiltonian, which automatically ensures the conservation of the phase space volume and the magnetic moment even in the approximate expressions obtained by truncating the perturbation expansion up to the finite order. In the gyrokinetic theory, the particle Hamiltonian (or the particle energy) is not an invariant since the fluctuating electromagnetic fields are treated. Instead, the conserved quantity is the total energy of the system, which is given by the sum of the kinetic energy of the particles and the energy of the electromagnetic fields. However, the proof of the total energy conservation is not trivial in the conventional formulation, where only the particle dynamics are described by the Hamiltonian or Lagrangian. Then, it seems natural that the formulation should be extended in order to derive governing equations for both the particles and the electromagnetic fields from the first principle. The purpose of the present work is to present such an extended formulation of the gyrokinetic theory [1].

The Lagrangian for the gyrokinetic Vlasov-Poisson-Ampère system takes the following form

$$L = \sum_a \int d^6\mathbf{Z}\; D_a F_a(\mathbf{Z},t) L_a(\mathbf{Z}_a, \dot{\mathbf{Z}}_a, t) + \frac{1}{8\pi}\int_V d^3\mathbf{x} \times \left(|\nabla\phi_1|^2 - |\nabla \times (\mathbf{A}_0 + \mathbf{A}_1)|^2 + \frac{2}{c}\lambda \nabla\cdot\mathbf{A}_1 \right)$$

where $\mathbf{Z}_a = (\mathbf{X}_a, U_a, \mu_a, \xi_a)$ represents the gyrocenter coordinates for species a obtained by the Lie-trasform method, D_a is the Jacobian, F_a denotes the distribution function, $(\phi_1, \mathbf{A}_1)$ are the potentials for the fluctuating electromagnetic fields, and the last term with λ is included to give the Coulomb gauge condition $\nabla\cdot\mathbf{A}_1 = 0$. The single-particle Lagrangian L_a for species a is given by

$$L_a = \frac{e_a}{c}\mathbf{A}_a^* \cdot \dot{\mathbf{X}}_a + \frac{m_a c}{e_a}\mu_a \dot{\xi}_a - H_a$$

with $\mathbf{A}_a^* = \mathbf{A}_0(\mathbf{X}_a) + \frac{m_a c}{e_a}U_a\mathbf{b} - \frac{m_a c^2}{e_a^2}\mu_a \mathbf{W}(\mathbf{X}_a)$ and the single-particle Hamiltonian

$$H_a = \frac{1}{2}m_a U_a^2 + \mu_a B_0(\mathbf{X}_a) + e_a\langle\psi_a\rangle + \frac{e_a^2}{2m_a c^2}\left\langle |\mathbf{A}_1(\mathbf{X}_a + \rho_a, t)|^2 \right\rangle - \frac{e_a}{2}\left\langle \left\{ \tilde{S}_{a1}, \tilde{\psi}_a \right\} \right\rangle$$

where $\psi_a = \phi_1 - \frac{1}{c}\mathbf{v}_{a0}\cdot\mathbf{A}_1$. Here, $\langle\cdot\rangle$ denotes the gyrophase average, and $\tilde{S}_{a1}$ represents the generating function for the gyrocenter symplectic Lie transform.

From the variational principle for the Lagrangian shown above, all the governing equations for the gyrokinetic system, the gyrokinetic equation for the particle distribution function and the gyrokinetic Poisson-Ampère equations for the electromagnetic fields, are derived. In this generalized Lagrangian formulation, the energy conservation property for the total nonlinear gyrokinetic system of equations is directly shown from the Noether's theorem. This formulation can be utilized in order to derive the nonlinear gyrokinetic system of equations and the rigorously conserved total energy for fluctuations with arbitrary frequencies. Simplified gyrokinetic systems of equations with the conserved energy are obtained from the Lagrangian with the small electron gyroradii, quasineutrality, and linear polarization-magnetization approximations. These simplified systems of equations, which retain the rigorous energy conservation, are considered to be useful for numerical simulation of plasma turbulence and anomalous transport.

References
1) Sugama, H. : Phys. Plasmas **7** (2000) 466.