§30. Onset of Turbulence in Minimal Plane Couette Flow

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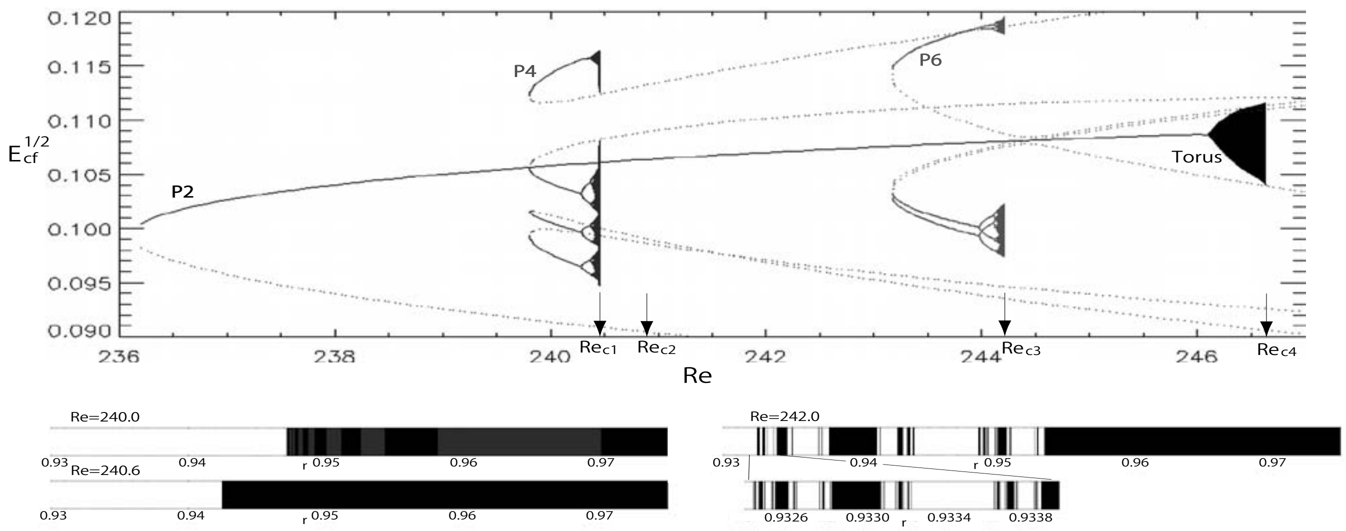
Transition to turbulence in wall-bounded flow such as pipe flow or plane Couette flow is essentially caused by finite amplitude disturbances, and has been one of the biggest open problem in fluid dynamics. Recent works by Kreilos & Eckhardt[1] and Avila et al.[2] showed numerically the onset of chaos in plane Couette flow and pipe flow respectively. In both works, besides periodicity, they imposed additional symmetry conditions and successfully obtained the stable nonlinear solutions, which are the upper-branches arising from saddle-node bifurcations to be the origin of chaotic attractors via period-doubling cascade or torus explosion. They also showed that the chaotic attractors lose their stability within narrow ranges of Reynolds numberRe.

Here we consider incompressible Newtonian flow in minimal plane Couette system with the domain size $1.755\pi*2*1.2\pi$. (for example, see [3]) No additional symmetry is imposed besides the periodic boundary conditions (unlike in [1] and [2]). We integrate numerically this system with spectral-Galerkin method and investigate final flow state for $236 \le Re \le 247$. Top figure shows bifurcation diagram of this system. Laminar flow(LF), which corresponds to $E_{\mathrm{cf}} = 0$, is linear stable. For $236.1 \le Re \le 246.6$, "several attractors coexist with LF". This coexistence makes it easy to find global changes in phase space. Beginning of the nonlinear solutions is the pair of periodic orbits (P2) caused by saddle-node bifurcation [3], and each of them has two local maxima per a cycle with same value. (This is same for P6.) The upper-branch of P2 ($\mathrm{UB_{P2}}$) loses stability at $Re = 246.1$ by supercritical Neimark-Sacker bifurcation [3] resulting in the stable torus. P4 and P6 also appear by saddle-node bifurcation and each of them leads to the chaotic attractor by period-doubling cascade.

We summarize below some important global bifurcation points. At $Re_{c1} = 240.4$ the boundary crisis occurs between $\mathrm{UB_{P2}}$ and the chaotic attractor, and the chaotic trajectory can approach to any neighborhood of the $\mathrm{LB_{P4}}$. Above this Re the chaotic attractor is replaced by the chaotic saddle and all trajectories from neighborhood of it are attracted to $\mathrm{UB_{P2}}$. This chaotic set also touches $\mathrm{LB_{P2}}$ at $Re_{c2} = 240.9$, and "the fractal basin boundary" appears between $\mathrm{UB_{P2}}$ and LF (bottom-right figure). Above Re_{c3} the chaotic attractor originated in P6 disappear and trajectories from the neighborhood of it go to LF. At Re_{c3} "the chaotic orbit touches the fractal basin boundary". Finally at Re_{c4} the collapse of the torus occurs through $\mathrm{LB_{P6}}$, and there is no attractor except for LF.

[1] T. Kreilos and B. Eckhardt, *Chaos*, **22**, 047505, 2012.
[2] M. Avila, F. Mellibovsky, N. Roland and B. Hof, *Phys. Rev. Lett.*, **110**, 224502, 2013.
[3] G. Kawahara, *Phys. Fluids*, **17**, 041702, 2005.



(Top) Bifurcation diagram of minimal plane Couette flow for $236 \le Re \le 247$. Lines and filled areas represent attractors and dotted lines are saddles. E_{cf} is local-maximum of the cross-flow energy, which is the same quantity used in Kreilos and Eckhardt[1]. At $Re = 236.1, 239.8$ and 243.2 periodic orbits are created by saddle-node bifurcation. During one cycle these have two, four and six local maxima of E_{cf} respectively and have stable upper branch (P2, P4 and P6). Major global bifurcation points are represented by allows at $Re_c = 240.4, 240.9, 244.2$ and 246.6. ($Bottom$)Basin of attractors along lines in phase space. White, Black and Blue represent basin of the origin (laminar flow), P2 and P4 respectively. The horizontal axises r denotes the distance from the origin.