§14. Quantum Conduction Phenomena in Hot Dense Plasmas

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Rapid electrical heating is an essential step in the production of hot dense plasmas in Z-pinch pulsed power machines. Laser heating of metallic targets can be regarded as Joule heating by the laser electromagnetic field and again the essential material property is (high-frequency) electrical conductivity. We have recently studied AC conductivity of free electrons in simple metals. While electrical conduction in hot plasmas is usually thought to be entirely classical, careful inspection shows there can be quantum effects at moderate plasma temperatures. These effects include stimulated emission, quantum statistics, and quantum minimum impact parameters resulting from interference in electron-ion collisions and in multiple scattering.

Plasma spontaneous emission of visible light has low intensity compared to focused laser radiation, but stimulated emission can be very important. Stimulated emission is a quantum effect which gives an apparent reduction of the absorption coefficient depending on the frequency ω and the plasma temperature kT:

$$\text{abs - stim emiss} = [1 - \exp(-\hbar\omega/kT)] * \text{absorption}$$

Here $\hbar$ is Planck's constant, the signature of a quantum effect. The stimulated emission factor [in braces] appears in some theories but not in others.

Hot plasmas are non-degenerate, and have a negative chemical potential. However electrons in normal metals are degenerate and obey Fermi-Dirac (quantum) statistics, required whenever kT is small compared to the Fermi energy μ_0. Plasmas at temperatures 1-20 eV are becoming nondegenerate.

The electron-ion Coulomb scattering cross-section diverges like $1/\theta^4$ for small deflection angles θ. When we calculate the total cross-section, weighting the scattering by electron momentum-loss, the integral diverges as $1/\theta$. This logarithmic divergence should be cut off at a minimum angle $\theta_{min} \sim b_{min} / b_{max}$. While b_{max} is a classical plasma screening length (Debye length for weakly-coupled plasma), $b_{min} = \hbar/mv$ is the quantum distance of closest approach. Quantum interference is the origin of this cutoff and brings a dependence upon the ion pair-distribution function at short distances.

Multiple scattering occurs when an electron has collisions with more than one ion during a time $1/\omega$. We do not have a satisfactory quantum calculation of this effect.

The simple classical formula for frequency-dependent conductivity is the Drude formula,

$$\sigma(\omega) = \frac{ne^2}{m} \left\langle \frac{\tau}{1 - i\omega\tau} \right\rangle \tag{1}$$

The real part of the conductivity leads to absorption (heating) and the imaginary part leads to refraction. The heating calculated from the Drude formula has no stimulated emission factor.

Absorption could also be calculated by using a result from the theory of radiation, the Kramers' cross-section for inverse-bremsstrahlung absorption:

$$\sigma^{IB}(\varepsilon, \hbar\omega) = \frac{8\pi^3}{3\sqrt{3}} Z^2 a_o^5 \left(\frac{e^2 / a_o}{\varepsilon}\right)\left(\frac{e^2 / a_o}{\hbar\omega}\right)^3 \tag{2}$$

This is the cross-section for an electron of energy ε to absorb a nearby photon of energy $\hbar\omega$ during collision with an ion of charge Ze ($a_o = \hbar^2/me^2$ = Bohr radius). Averaged over the electron velocity distribution Kramers' formula leads to an absorption rate equivalent to a "quantum" conductivity,

$$\mathrm{Re}\Big[\ \sigma(\omega)\ \Big] = \frac{4}{3\sqrt{3}} \frac{Z^2 e^6}{\hbar(\hbar\omega)^3}\, n_i kT \,\log\left(\frac{1 + e^{\mu/kT}}{1 + e^{(\mu - \hbar\omega)/kT}}\right)$$

The coefficients in front of the logarithm can be re-arranged to look like averages of the electron-ion collision rate in order to facilitate comparison with the Drude formula.

We examine this result in several limits. First, in the non-degenerate case a straightforward series expansion gives

$$\mathrm{Re}\Big[\ \sigma(\omega)\ \Big] = \frac{n_e e^2}{m} \frac{1}{\omega^2 \tau_B} \left[\frac{kT}{\hbar\omega}(1 - e^{-\hbar\omega/kT})\right] \tag{3}$$

Here τ_B is the relaxation time obtained from the Coulomb cross-section with the high-frequency Coulomb logarithm. The quantity before the brackets is the high-frequency ($\omega\tau >> 1$) limit of the Drude formula, but Eq. (3) explicitly shows a stimulated emission factor which can significantly change the result from the Drude form.

In the limit $\hbar\omega << kT$, the factor in brackets is unity and the Drude formula applies. However, the condition $\hbar\omega << kT$ is not necessarily satisfied for plasmas in the temperature range .1 eV - 10 eV.

An important case is the degenerate (Fermi-Dirac) case, and in this case, for $\hbar\omega << \mu_0$, we obtain

$$\mathrm{Re}\Big[\ \sigma(\omega)\ \Big] = \frac{n_e e^2}{m} \frac{1}{\omega^2 \tau_B} \tag{4}$$

In this formula τ_B is again the collision time. It is quite surprising that in this case there is no stimulated emission factor: the quantum theory leads to the Drude result after all!

Eq. (4) explains the success of the Drude formula for electrical engineering with metallic conductors. The theory based on Eq. (1) also succeeds to describe short-pulse laser absorption by aluminum targets. However it is not true that quantum theory always gives the Drude result, as Eq. (3) has shown. For degenerate matter with $\hbar\omega >> \mu_0$ there is also a strong quantum correction to the Drude result.