

§13. Hydrogen Spectra in Microwave Fields

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Hydrogen spectra from fusion machines do not always show the expected intensities for Zeeman split lines. Such spectra have been collected by Fujimoto and Goto.[1] Such spectra may be due to improper alignment of the spectrometer or to effects of reflected light. Is there another process able to produce anomalous Zeeman spectra? We examine the possible perturbation of hydrogen spectra by high-power microwaves.

In certain experiments, several Megawatts of 82.6 GHz electron cyclotron resonance heating is pumped into the LHD machine. This intense microwave radiation is nearly resonant with the hydrogen Zeeman splitting in a 6T field. We performed a numerical calculation to understand the effect of this radiation.

The calculation uses one-electron basis states with quantum numbers n, ℓ, m, σ . The isolated-atom Hamiltonian H_0 includes the non-relativistic binding energy and fine structure energy. Static electric and magnetic fields give a perturbation $H_S = -\boldsymbol{\mu} \cdot \mathbf{B}_0 - e\mathbf{r} \cdot \mathbf{E}_0$, where $\boldsymbol{\mu} = \boldsymbol{\mu}_0 \mathbf{L} + 2\boldsymbol{\mu}_0 \mathbf{S}$ is the sum of orbital and spin magnetic moments ($\boldsymbol{\mu}_0 = e\hbar/2mc$ is the Bohr magneton). The dynamic perturbation $H_d(t)$ has the same form as H_S but uses the time-dependent electric and magnetic fields $\mathbf{E}(t)$, $\mathbf{B}(t)$ of the microwave heating. The Hamiltonian is not diagonal. In our code, we can use two sets of basis states: a large set, 1s through 4f, with spin (60 states), and a small set, 2s through 3d, without spin (13 states).

The code contains matrices for $X, Y, Z, L_x, L_y, L_z, S_x, S_y, S_z, H(t)$, and ρ_0 where $H(t)$ is the total Hamiltonian and ρ_0 is the (initial) density matrix. The code solves the time-dependent Schroedinger equation:

$$i\hbar \frac{d}{dt} U(t) = H(t) \circ U(t) \quad (1)$$

$H(t)$ has the time-dependence of the microwave electric and magnetic fields. From $U(t)$, the spectrum is calculated by forming the dipole autocorrelation function,

$$\Phi_x(t) = \text{Tr} \left[\hat{x} \hat{U}^+(t) \hat{x} \hat{U}(t) \hat{\rho}_0 \right] \quad (2)$$

The emission spectrum [2] is:

$$P(\omega) = \frac{2e^2 \omega^4}{3c^3} \text{Re} \int_0^\infty e^{i\omega t} e^{-\lambda t} [\Phi_x(t) + \Phi_z(t)] dt \quad (3)$$

Eq. (3) gives the radiation seen from the y-direction having electric vector in the x or z directions; the polarizations are easily separated.

We evaluate $\Phi(t)$ for times up to ~ 40 psec in order to resolve Zeeman splittings, but require a time-step $dt \sim .4 \cdot 10^{-17}$ seconds to resolve the K-shell. This means the code must take 10 million time-steps. For stability and accuracy, several matrix operations are needed for each time-step. Therefore, it is important to carefully organize the calculation; we use various numerical tricks and use the small set of states for most calculations. We collected the necessary atomic data and checked it for consistency.

There are several ways to use the code. We can calculate the effect of microwaves on Zeeman or Stark spectrum. We are most interested to see if there is a new way to measure the local microwave intensity in the plasma.

The code can easily calculate the DC Stark effect, DC Zeeman effect, Stark-Zeeman mixtures, or the spectrum from aligned or polarized input populations. In these cases there are only static electric or magnetic fields.

The code can also calculate harmonic production in laser interaction with H-like ions, and can calculate cross-sections for excitation by the pulsed microfield $\mathbf{E}(t)$ which arises in an ion-ion collision.

The calculations performed so far show that the code reproduces the expected Stark effect in a strong DC electric field, gives the expected Zeeman effect (σ, π components) for static B-field and show that alignment changes ratio of these components. For an oscillating electro magnetic field of resonant frequency having microwave power $\sim 30 \text{ kW/cm}^2$, we find a 6% change in the Zeeman profile. This power is in the range of the intensities that exist in some fusion machines.

References:

- (1) Professor T. Fujimoto and Dr. M. Goto, unpublished communication.
- (2) H. Griem,, **Principles of Plasma Spectroscopy**, Cambridge University Press, Cambridge, 1997.