§26. A Study of Sub-grid-scale Model for Extended Magnetohydrodynamic Turbulence

Miura, H., Araki, K. (Okayama Univ. Sci.),
Hamba, F. (Univ. Tokyo), Yokoi, N. (Univ. Tokyo)

A sub-grid-scale (SGS) model for extended magneto-hydrodynamic (MHD) turbulence is studied for the purpose of applying it to a torus-plasma simulation with the two-fluid (Hall) and the gyro-viscous term. Since SGS models for a high Lundquist number MHD turbulence are not well established, we assume as the first step homogeneous and isotropic turbulence and incompressibility of fluids to avoid complexity of modeling. Then the gridi-scale (GS) MHD equations can be expressed as

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} = -\nabla\cdot\left(\overline{\mathbf{u}}\,\overline{\mathbf{u}} - \overline{\mathbf{B}}\,\overline{\mathbf{B}}\right) - \nabla\left(\overline{p} + \frac{1}{2}\left|\overline{\mathbf{B}}\right|^2\right) + \nu\nabla^2\overline{\mathbf{u}} + \nabla\cdot\left[\left(\overline{\mathbf{u}}\,\overline{\mathbf{u}} - \overline{\mathbf{u}\mathbf{u}}\right) - \left(\overline{\mathbf{B}}\,\overline{\mathbf{B}} - \overline{\mathbf{B}\mathbf{B}}\right)\right], \qquad (1)$$

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = -\nabla\times\left[\overline{\mathbf{u}}\times\overline{\mathbf{B}} - \left(\overline{\mathbf{u}}\times\overline{\mathbf{B}} - \overline{\mathbf{u}\times\mathbf{B}}\right) + \eta\,\overline{\mathbf{j}}\right] + \varepsilon_H\nabla\times\left[\overline{\mathbf{j}}\times\overline{\mathbf{B}} - \left(\overline{\mathbf{j}}\times\overline{\mathbf{B}} - \overline{\mathbf{j}\times\mathbf{B}}\right)\right] \qquad (2)$$

$$\overline{\mathbf{j}} = \nabla\times\overline{\mathbf{B}}, \qquad (3)$$

where $\overline{\mathbf{B}}, \overline{\mathbf{u}}$ are the magnetic and the velocity field vector, respectively. The last term in the right-hand-side in eq.(2) represents the Hall term and the SGS contribution to the GS components of the Hall term. The overbar represents the GS component, which is obtained by operating a low-pass filter to the original variable. Here we assume the low-pass filter is the sharp filter of the cut-off wave number k_c in the Fourier space.

We adopt the SGS model developed by Hamba and Tsuchiya [1], which can be expressed as

$$\nabla\cdot\left[\left(\overline{\mathbf{u}}\,\overline{\mathbf{u}} - \overline{\mathbf{u}\mathbf{u}}\right) - \left(\overline{\mathbf{B}}\,\overline{\mathbf{B}} - \overline{\mathbf{B}\mathbf{B}}\right)\right] = \nabla\cdot\left[\nu_{SGS}\overline{S}_{ij}\right] \qquad (4)$$

$$\overline{S}_{ij} = \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}, \qquad (5)$$

$$\nu_{SGS} = C_\nu\Delta^2\left(\frac{1}{2}C_\nu\overline{S}_{ij}^{\,2} + C_\lambda j_i j_i\right)^{1/2}, \qquad (6)$$

$$\left(\overline{\mathbf{u}}\times\overline{\mathbf{B}} - \overline{\mathbf{u}\times\mathbf{B}}\right) = C_\lambda\Delta^2\left(\frac{1}{2}C_\nu\overline{S}_{ij}^{\,2} + C_\lambda j_i j_i\right)^{1/2}. \qquad (7)$$

Firstly we have verified that the SGS model works well for reproducing the energy spectra of MHD and Hall MHD turbulence. By carrying out LESes with the number of grid points 64^3 and comparing the energy spectra to those obtained by the direct numerical simulations (DNSes) with the number of grid points 512^3, we can see that both the kinetic and the magnetic energy spectra of LESes coincide well with those of DNSes.

Secondly, we projected the DNS data to the model expressed in eqs.(4)-(7). As the first step, we compute the right-hand-side of eqs.(4) and (7). We tentatively call the two computations as "modeled fields". Next, we project the left-hand-side of eqs. (4) and (7) to the modeled field. The squared residue of the projection to the modeled field In Fig. 1 and 2, the residues of the projection for the induction term and the Hall in eq.(4) are shown, respectively. In Fig.1, the residues are compared between Hall MHD turbulence and single-fluid MHD turbulence. Fig.1 suggests that the model works less effectively in Hall MHD turbulence than in single-fluid MHD turbulence. In Fig.2, the residues are compared among various resolutions by applying the sharp low-pass filter to the DNS data.Fig.2 suggests that the Hall term does not necessarily respond to the low-pass filter, suggesting that the applicability of the Smagorinsky-type model to the Hall term is quite limited. However, we have found recently that there can be a threshold of the resolution (cut-off wave number of the low-pass filter) in a higher wave number region, by analyzing DNS data of 1024^3 grid points. When the cut-off wave number is increased, the residue becomes smaller drastically. This suggests that the Smagorinsky-type model can be applicable when a very high-wave-number components of the Hall term is replaced by the SGS model. Further studies are continued to enable an LES of the extended MHD model.

[1]Hamba F. and Tsuchiya M. 2010, Phys. Plasmas 17, 012301.

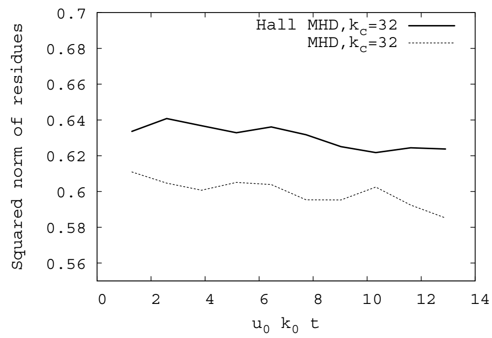

Fig.1: Residue of the projection of the induction term in eq.(2) obtained by DNS to the SGS model (7).

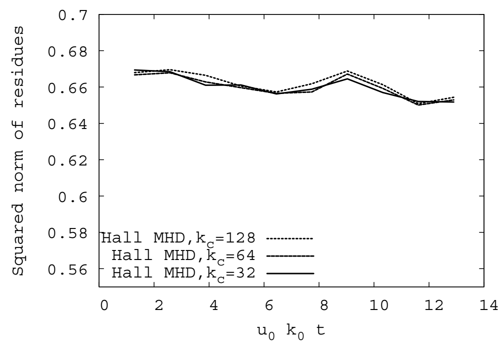

Fig.2: Residue the projection of the Hall term in eq.(2) obtained by DNS to the SGS model (7).