§36. A δf Monte Carlo Method to Evaluate the Bootstrap Current with Moment-equation Approach

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The δf Monte Carlo method has been used as an efficient technique to evaluate the so-called bootstrap current coefficient D_{31} by guiding-center particle simulations based on the pitch-angle scattering approximation. For example, Isaev et al. applied the VENUS+δf code[1] to the calculation of D_{31} for the LHD device. However, to predict experimentally observed bootstrap current, we also need to take into account the momentum conservation of like-particle collisions and the coupling between electrons and ions. To date, such a self-consistent Monte Carlo simulation has not been reported.

To overcome the above difficulties, we developed a method to calculate D_{33} in addition to D_{31} using the δf method.[2] According to the moment-equation approach,[3] the extension to the calculation of D_{33} enables us to determine a closure of moment equations, more specifically, the viscosity-flow relations. It thus allows us to evaluate the net bootstrap current from algebraic solutions of moment equations. This approach takes the computational advantages of the pitch-angle scattering approximation and yet satisfies the conservation laws of collision operators.

We briefly outline δf weighting scheme developed in this work.[2] We begin with the following two drift-kinetic equations,

$$(V_{\|}\text{-}C)\, g_1 = -\,\sigma_1^{+}, \qquad (1)$$
$$(V_{\|}\text{-}C)\, g_3 = -\,\sigma_3^{+}. \qquad (2)$$

To solve these equations, we prepare two marker-weights $w_1 = g_3/F_M$ and $w_3 = g_3/F_M$, where F_M is a marker distribution function. The two mono-energetic transport coefficients D_{31} and D_{33} were independently calculated from the Monte Carlo integration of parallel velocity weighted by w_1 and w_3, respectively. These can be formally written in terms of the inner-product operation such that

$$D_{31} = -\,(\nu B\xi, g_1), \qquad (3)$$
$$D_{33} = -\,(\nu B\xi, g_3). \qquad (4)$$

It is worth to comment that the choice of eqs. (1)-(4) was crucial not only for theoretical but also for numerical reasons.

Using these two coefficients, two viscosity coefficients M and N can be obtained, which correspond to the damping and driving rate of neoclassical flows. The normalized viscosity coefficients[3] are given by $M^*=(\nu_D/\nu)^2 D_{33}^*/[1-(3/2)(\nu_D/\nu) D_{33}^*/\langle B^2\rangle]$, $N^*=(\nu_D/\nu) D_{31}^*/[1-(3/2)(\nu_D/\nu) D_{33}^*/\langle B^2\rangle]$. These coefficients have been evaluated only by the DKES code so far.

We have implemented the technique developed here to the VENUS+δf code, and tested the calculation of viscosity coefficients for simple model heliotrons $B = B_0[1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(2\theta - 10\zeta)]$. We illustrate numerical results in Fig. 1 and Fig. 2. For comparison, we have also plotted asymptotic solutions and the results obtained from the DKES code.[4] The results calculated with the VENUS+δf code are in reasonable agreement with these reference data.

In summary, the δf method developed here can allows us evaluate the bootstrap current quantitatively with simple modification to the existent Monte Carlo code such as the VENUS+δf code. The present work will be useful for quantitative analysis of the bootstrap current in LHD on the basis of neoclassical theory.

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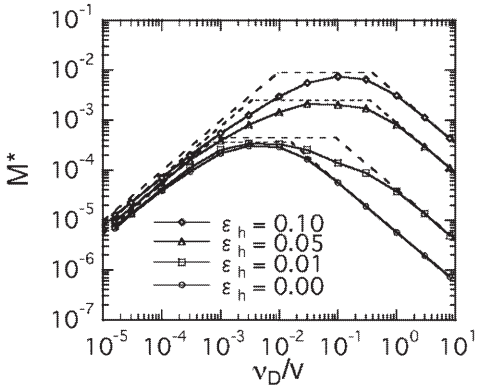

Fig. 1. The normalized viscosity coefficient M^* vs. collisionality calculated with VENUS+δf. We have also plotted the asymptotic solutions for characteristic collisionality regimes.

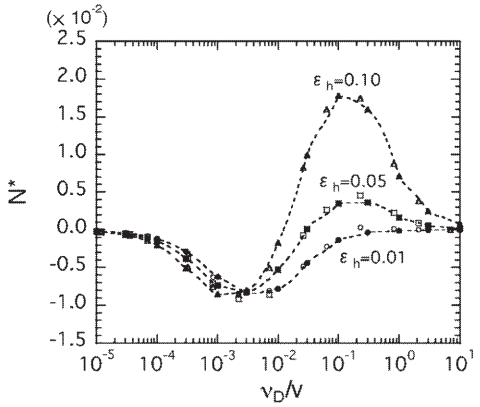

Fig. 2. Comparison of the normalized viscosity coefficients N^* between DKES (closed symbols) and VENUS+δf (open symbols).

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2) Matsuyama, A. et al.: Phys. Plasmas **16** (2009) 052501
3) Sugama, H. and Nishimura, S.: Phys. Plasmas **9** (2002) 4637
4) van Rij, W. I. and Hirshman S. P.: Phys. Fluids **1** (1989) 563