§32. A Signature of Wave Collapse in the GNLS Model of Plasma Turbulence

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A term "wave collapse" is used to describe the formation of the singularity in a finite time in mathematical models describing nonlinear wave systems. It is one of the basic phenomena in nonlinear physics. The singularity also signals the limit of the model validity. It was predicted that the presence of the nonlocal nonlinearity eliminates collapse in the system governed by a nonlocal NLS equation.[1-5]

We discuss the wave collapse existence in the system described by the generalized nonlinear Schrödinger (GNLS) type of equation with two additional nonlocal nonlinear terms [2]:

$$i\frac{\partial A}{\partial t}+\frac{1}{2}A_{xx}+\frac{3}{16}|A|^2A-\frac{1}{8}(|A|^2)_{xx}A+\frac{1}{48}(A^2)_{xx}A^*=0, \quad (1)$$

where A is vector potential envelope. This equation models nonlinear coherent structures in, e.g.: ETG turbulence [5] and weakly relativistic laser-plasma interaction [2-4].

Localized stationary solution of (1) is found in a form of a moving soliton with 3 conserved quantities[3]:

$$A=\rho(u)\exp[i\theta(u)+i\lambda^2 t], \quad (2)$$

where $u=x-vt$, and v is the soliton velocity. After introducing the ansatz (2) in Eq.(1), first integration for localized boundary conditions ($\rho(u),\rho(u)_u,\rho(u)_{uu}\to 0$ for $u\to\pm\infty$) gives the soliton amplitude equation:

$$(\rho_u)^2=\left(2\lambda^2-v^2-\left(\frac{3}{16}-\frac{v^2}{12}\right)\rho^2\right)\rho^2\Big/\left(1-\frac{5}{12}\rho^2\right). \quad (3)$$

Singular point is $\rho_c=\sqrt{12/5}$, while region $\rho<\rho_c$ corresponds to a bright soliton. Additional integration of (3) yields a moving soliton solution in implicit form, with the maximum soliton amplitude $\rho_0^{\,2}=(2\lambda^2-v^2)/(3/16-v^2/12)$. For $\rho_0 << \rho_c$ the soliton profile is secant hyperbolic, like the soliton of the standard cubic NLS equation. When ρ_0 approaches the value of ρ_c, the soliton profile steepens and transits toward the pointed, cusp type of a profile (Fig.1).

Stability analysis by using Vakhitov-Kolokolov stability criterion shows that moving EM solitons are stable in the region $\lambda<\lambda_s$, where λ_s corresponds to the maximum value of photon number (wave energy) for a given velocity. A set of direct numerical simulations of the nonlinear model (1) has been performed in order to investigate soliton dynamics [3]. Initially perturbed stable solitons exhibit long lived oscillating behavior of the breather type, with the amplitude excursion from the initial value increasing as the level of perturbation grows, eventually leading to a rapid aperiodic growth of the amplitude. This continues up to the point when the amplitude reaches the critical value, creating a highly unstable cusp structure. Due to the coincidental break up of the numerical scheme, we were unable to follow the dynamics of this structure further.

The process above is similar to the initial stage of the collapse phenomenon, which is predicted in the case of the GNLS equation [4]. To conclude if the unstable cusp soliton structure in above model will collapse or break up, we have attempted some further analyses:

1) *Bifurcation analysis*: This approach is analogous to the one of Ref. [6]. Stability analysis shows that depending on parameters λ, v two types of bifurcation are possible: tangential bifurcation that corresponds to stable soliton solutions, and Hopf-Andronov (HA) bifurcation. The cascade of HA bifurcations can be the origin of complex behavior [7] of our system (collapse as one of possible scenarios). However the additional studies are needed to make a definite conclusion about the collapse.

2) *Linear stability analysis* around the cusp solution: Similar to Ref. [8], analysis of the simplified system (without the last term in Eq.(1)) is performed. It is shown that different families of solution are possible depending on the initial conditions and system parameters. One of them is the localized mode of a cusp type, as found in our model. This mode reminds of the so-called nonlinear explosion mode [8]. However, the strict conclusion about the collapse of the full system (1) is still missing.

3) *Virial theorem*: In the case of the NLS equation (first three terms in (1)), Virial theorem gives a sufficient condition for the collapse and an estimate of the collapse threshold [8]. However, we cannot reach that kind of conclusion, due to a singularity at the critical amplitude (3).

At present, the issue of wave collapse in GNLS model remains unresolved. One of the possible solutions is the inclusion of the thermal and higher order dissipative effects in the model. These effects will eventually saturate the system and remove a mathematical singularity to fill the gap in our understanding of its long time behavior. However turbulence model based on (1) may not be well founded [5]

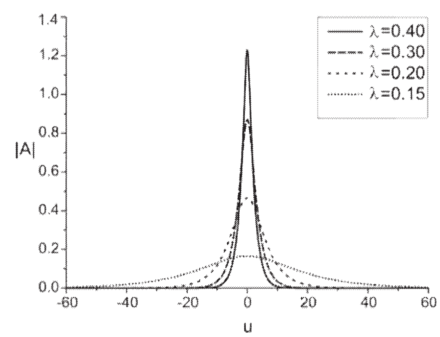

Fig.1. Soliton profile steepening (v=0.2)

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