§ 27. Reynolds Number Dependence of Low-Pressure Vortex in Isotropic Turbulence

Makihara, T., Kida, S.

Turbulence is an assembly of various kinds of vortices. Among others the tubular swirling motion of concentrated vorticity is observed ubiquitously. They play central roles in turbulence dynamics. Understanding of these vortical structures may therefore be useful for the prediction and control of turbulence.

An objective eduction scheme of tubular swirling vortices has been recently introduced and developed, which is called the low-pressure vortex method[1)]. This enables us to estimate the physical characteristics of tubular vortices quantitatively. It was found that the mean diameter of tubular vortices is 10 times Kolmogorov length and they are rotating with 3 times Kolmogorov velocity[2)]. High vorticity is concentrated around the cores and surrounding double spiral layers. The axial component of vorticity is dominant in the core, whereas the perpendicular component in the layers[3)]. The above results, however, were derived from numerical turbulence of Reynolds number less than 200. It is interesting that for larger Reynolds numbers the tubular structure remains to exist stably and to play important roles in turbulence dynamics.

We consider here the motion of an incompressible viscous fluid which is governed by the continuity equation and the Navier-Stokes equation. The flow is confined in a periodic cube of side length 2π. The velocity and the pressure fields are expanded in the Fourier series. The above equations are solved numerically by the spectral/Runge-Kutta-Gill scheme on 256^3 or 512^3 grids. The external force is supplied to maintain a statistically steady state. The Reynolds numbers R_λ of the isotropic turbulence obtained by these calculations are 82, 123 and 174, where R_λ is defined by Taylor microscale λ, RMS velocity u_{RMS} and kinematic viscosity ν. Note that the cut-off wavenumber $k_{\max}\eta$ is taken to be common to all the three cases so that the numerical accuracy may be comparable with each other, where η denotes the Kolmogorov length. Tubular vortices in each isotropic turbulence are identified by low-pressure vortex method.

In the present method the position of all nodes of vortex axes are recorded as three coordinates, the direction of segments between successive nodes as three-dimensional vectors, and the core boundary surfaces as the coordinates of 32 points around all the nodes. These data enable us to calculate the diameter and the circulation of vortices. The volume occupied by vortex cores may be estimated by counting the number of grid points inside core boundaries. At the same time the contribution from the vortex cores to the enstrophy and energy-dissipation rate is calculated. The statistical quantities thus obtained are shown in Table 1. These are the mean values averaged over 23, 17 and 17 snapshots in the respective cases. Here, D denotes the diameter, V the volume, Γ the circulation, u_θ the swirling velocity on the core boundary, u_η the Kolmogorov velocity, $\omega_{\|}$ the axial velocity on the vortex axes, and l_{TOTAL} the total length of vortex axes. Subscript 'core' indicates the contribution from the vortex core. It is seen that the low-pressure vortices have the mean diameter of 10η and swirling velocity of $3u_\eta$, and occupy 30% volume of the flow field. Their contributions to the enstrophy and energy dissipation are 60% and 30% of the totals, respectively. Although the magnitude of the vorticity has no direct connection with the definition of the low-pressure vortex, the axial vorticity is larger than the RMS vorticty by factor 2.5 or more.

In order to examine the Reynolds-number dependence of these quantities we assume they scale as $R_\lambda{}^\alpha$ and estimate the exponent α by use of the data at three different values of the Reynolds number. The results are listed at the bottom of Table 1. As the Reynolds number increases, the volume occupied by vortices, the swirling velocity normalized by the RMS velocity decrease, the vortex Reynolds number Γ/ν, the swirling velocity normalized by the Kolmogorov velocity, the axial vorticity and the total length increase, whereas the diameter and the contribution of vortex cores to the enstrophy and energy-dissipation rate are invariant.

The present results suggest that the vortices remain to exist stably and play important roles in turbulence dynamics even at larger Reynolds numbers.

Reference
1)S. Kida and H. Miura, Eur. J. Mech. B/Fluids **17**, (1998) 471.
2)S. Kida, Proc. ICTAM2000, Kluwer, (2001) 445.
3)S. Kida and H. Miura, *J. Phys. Soc. Japan*, **69**, (2000) 3466.

Table 1 Core statistics

R_λ	$\frac{D_{\mathrm{core}}}{\eta}$	$\frac{V_{\mathrm{core}}}{V}$	$\frac{\Gamma}{\nu}$	$\frac{Q_{\mathrm{core}}}{Q}$	$\frac{\epsilon_{\mathrm{core}}}{\epsilon}$	$\frac{u_\theta}{u_\eta}$	$\frac{u_\theta}{u_{\mathrm{RMS}}}$	$\frac{\omega_{\|}}{\omega_{\mathrm{RMS}}}$	$\frac{l_{\mathrm{TOTAL}}}{L}$
82	10.44	0.348	109.5	0.603	0.314	2.975	0.648	2.48	405
123	9.93	0.298	107.4	0.566	0.277	3.001	0.533	2.64	945
174	10.76	0.305	123.5	0.585	0.308	3.262	0.487	2.71	2160
α	0.03	-0.19	0.15	-0.05	-0.05	0.12	-0.38	0.12	2.15