§10. Hybrid Method of Semi-Lagrangian and Additive Semi-implicit Runge-Kutta Schemes for Gyrokinetic Vlasov Simulations

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A hybrid method of semi-Lagrangian and additive semi-implicit Runge-Kutta schemes is developed for gyrokinetic Vlasov simulations in a flux tube geometry. The time-integration scheme is free from the Courant-Friedrichs-Lewy condition for the linear advection terms in the gyrokinetic equation. The new method is applied to simulations of the ion-temperature-gradient instability in fusion plasmas confined by helical magnetic fields, where the parallel advection term severely restricts the time step size for explicit Eulerian schemes. Linear and nonlinear results show good agreements with those obtained by using the explicit Runge-Kutta-Gill scheme, while the new method substantially reduces the computational cost [1)].

In order to overcome time-step-size restrictions from linear terms in the gyrokinetic equations, we apply a semi-Lagrangian scheme to the parallel advection and the mirror force terms, and deal with the magnetic drift term implicitly by the use of an operator splitting scheme. Implementing the hybrid method of semi-Lagrangian and additive semi-implicit Runge-Kutta schemes (SLASIRK) to the GKV code, we carried out linear and nonlinear simulations of the ITG instability in helical plasmas. Figure 1 plots the norm of the errors of the eigenfunction $L_{\rm E}$, the errors of the linear growth rate L_γ and the real frequency L_ω for the linearly most unstable mode as a function of the time step size. The results of $L_{\rm E}$ shows SLASIRK has the second-order temporal accuracy as expected, while L_γ and L_ω are more quickly converged. The norm of errors tends to approach a certain constant value for $\Delta t < 0.02 L_n/v_{\rm ti}$, where spatial errors dominate temporal ones. We note that explicit Eulerian schemes require time steps smaller than $0.005 L_n/v_{\rm ti}$ for stable computations.

Figure 2 shows elapsed time for a hundred simulation steps by employing the forth-order Runge-Kutta-Gill scheme (RKG) and SLASIRK. The amount of computations involved in the second-order SLASIRK is less than that in the fourth-order RKG, since the former computes the Poisson solver and nonlinear term twice per time step and the latter does four times. Excluding data input and output, RKG and SLASIRK take 7.79s and 4.45s per time step, respectively.

The newly-developed method allows twenty (three) times larger time step size for linear (nonlinear) runs than the original GKV which employs the forth-order explicit Runge-Kutta-Gill method with enough accuracy. By combining a large time step size and short elapsed time per time step, the efficiency of the gyrokinetic Vlasov simulation is largely enhanced by utilizing the present method.

1) Maeyama, S., Ishizawa, A., Watanabe, T.-H., Nakajima, N., Tsuji-Iio, S., Tsutsui, H. : Computer Physics Communications **183**, (2012) in press.

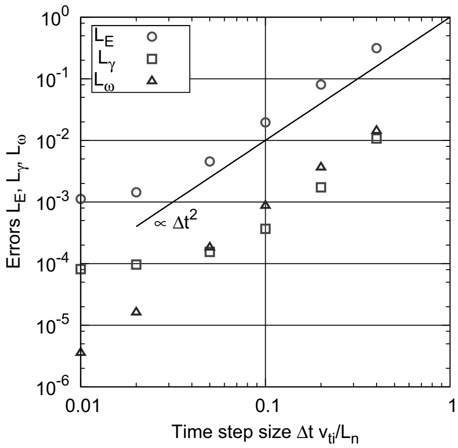

Fig. 1: The errors of the eigen function $L_{\rm E}$, growth rate L_γ and real frequency L_ω plotted as a function of the time step size Δt are shown by circle, square and triangle dots, respectively (where $k_y\rho_{\rm ti} = 0.417$ and SLASIRK is employed). The solid line represents a reference slope for the second-order accuracy.

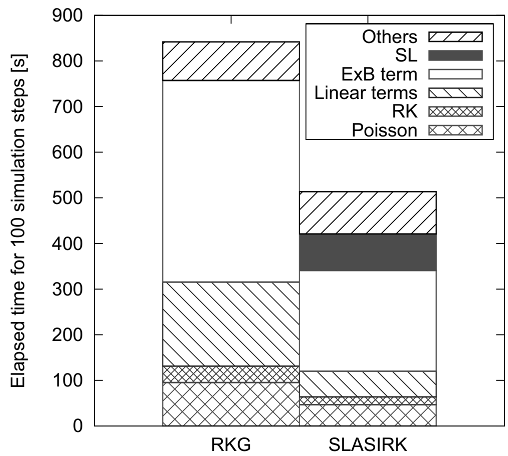

Fig. 2: Comparison of elapsed time between the fourth-order RKG and the second-order SLASIRK for a hundred simulation steps. From the bottom, boxes represent elapsed time to compute the Poisson solver, time integration by RKG or ASIRK, linear terms, nonlinear $\boldsymbol{E}\times\boldsymbol{B}$ term, time integration by semi-Lagrangian schemes, and the others including data input and output.