

## §19. A Numerical Method for Parallel Particle Motions in Gyrokinetic Vlasov Simulations

Maeyama, S., Tsuji-Iio, S., Tsutsui, H. (Tokyo Institute of Technology),  
Ishizawa, A., Watanabe, T.-H., Nakajima, N.

A semi-Lagrangian scheme is applied for the first time to computations of charged particle motions along magnetic field lines, to numerically solve the  $\delta f$  gyrokinetic equations in a flux tube geometry. This new solver adopted in the gyrokinetic Vlasov simulations has an advantage over the conventional Eulerian codes in calculating the parallel dynamics, because semi-Lagrangian schemes are free of the Courant-Friedrichs-Lewy (CFL) condition that restricts the time step size. A study of the accuracy of the parallel motion simulations reveals that numerical errors mainly stem from spatial (not temporal) discretization for realistic values of the grid spacing and time step, and it demonstrates the advantage of the semi-Lagrangian scheme. This novel numerical method is successfully applied to linear gyrokinetic simulations of the ion temperature gradient instability, where time steps larger than those restricted by the CFL condition can be employed <sup>1)</sup>.

The parallel dynamics of the perturbed ion distribution function  $f$  in the gyrokinetic Vlasov equation is given by

$$\frac{\partial f}{\partial t} + \{f, K\}_{\parallel} = 0, \quad (1)$$

where  $K = v_{\parallel}^2/2 + \mu B/m_i$  is the particle kinetic energy per unit mass and  $\{f, g\}_{\parallel} = \nabla_{\parallel} f \partial_{v_{\parallel}} g - \partial_{v_{\parallel}} f \nabla_{\parallel} g$  denotes the parallel Poisson brackets. Equation (1) expresses that the parallel dynamics is regarded as an advection of a distribution function along equi-contour lines of the particle kinetic energy  $K$  in the parallel phase space  $(z, v_{\parallel})$  shown in Fig. 1 (a). The contours with  $K < \mu B_0/m_i$  correspond to trapped particle trajectories, and the others represent trajectories of passing particles. Time integration of Eq. (1) is successfully computed by means of a semi-Lagrangian scheme with the second-order operator splitting method. The snapshots

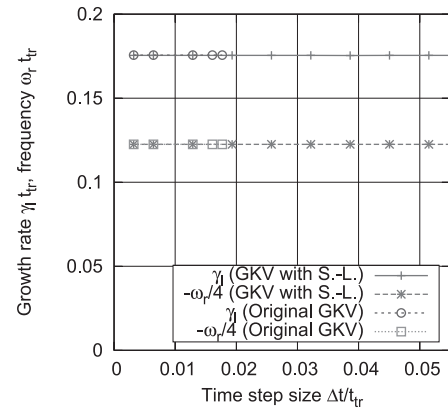


Fig. 2: Linear growth rates  $\gamma_i$  and real frequencies  $\omega_r$  as a function of the time step size  $\Delta t$ . Results using the semi-Lagrangian scheme for the parallel dynamics are compared with the original GKV results.

of equi-contours of the distribution function are shown in Fig. 1 (b)-(d). The ballistic motion of passing particles elongates the profile and makes fine-scale structures.

By implementing the semi-Lagrangian scheme to the GKV code <sup>2)</sup>, tests of a gyrokinetic simulation of the linear ITG instability are carried out for the Cyclone DIII-D base case parameter set. Figure 2 plots the linear growth rate and real frequency as a function of the time step size for a low- $k_y$  mode, where the CFL restriction due to the parallel motions is more severe than that due to the perpendicular drift motions in the original Eulerian GKV code. The results agree well with those obtained by the original GKV code: their relative errors are less than 0.1%. Therefore, the new solver with the semi-Lagrangian scheme enables us to take larger time step sizes than the CFL restriction and still achieve accurate results.

- 1) Maeyama, S., Ishizawa, A., Watanabe, T.-H., Nakajima, N., Tsuji-Iio, S., Tsutsui, H. : Plasma and Fusion Research **6**, (2011) to be published.
- 2) Watanabe, T.-H., Sugama, H., : Nuclear Fusion **46**, 24 (2006).

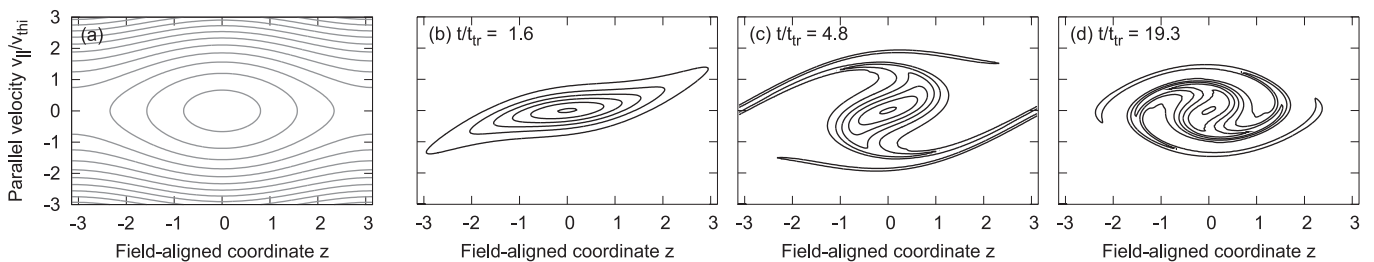


Fig. 1: (a) Contour lines of the particle kinetic energy  $K$ , and (b)-(d) snapshots of equi-contours of the distribution function  $f$  in parallel phase space. Horizontal and vertical axes are defined by the field-aligned coordinate  $z$  and the parallel velocity  $v_{\parallel}$ , respectively.