§4. The Construction of Generalized Magnetic Coordinates

M. Kurata (Dept. of Energy Engineering and Science, Graduate School of Engineering, Nagoya Univ.)
J. Todoroki

The magnetic coordinates are widely used in the study of the MHD equilibrium and stability in the toroidal plasma when the nested magnetic surfaces exist. Unfortunately, the good magnetic surfaces exist only in the limited region of torus and even inside the outermost magnetic surface there might exist complicated island structure. In such cases, the utilization of the conventional magnetic flux coordinates is not expected.

The Generalized Magnetic Coordinates (GMC) are the new one to supplement the magnetic coordinates system adequate to treat the general magnetic configurations. The GMC can be constructed in the region without nested magnetic surface and outside the outermost magnetic surface.

In GMC (ξ,η,ζ) the magnetic field is expressed in the form

$$\mathbf{B} = \nabla\Psi(\xi,\eta,\zeta)\times\nabla\zeta + H^{\zeta}(\xi,\eta)\nabla\xi\times\nabla\eta.$$

$H^{\zeta}=\sqrt{g}\,B^{\zeta}$ does not depend on ζ, here $\sqrt{g}$ is Jacobian. When the good magnetic surface exists, Ψ becomes independent of ζ and $\Psi(\xi,\eta)=$ Const. is the magnetic surface. The ζ-dependent part of Ψ corresponds to the destruction of the magnetic surface.

In order to check the GMC algorithm based on the transformation rule of the vector potential, the general method to construct a GMC is applied to the simple periodic model magnetic field.[1] The model magnetic field is ABC(Arnol'd-Beltrami-Childress) magnetic field in the Cartesian coordinates added constant magnetic field in the direction of z as follows.

$$\begin{cases} B_X = b\cdot\cos(2\pi y) + c\cdot\sin(2\pi z) \\ B_Y = c\cdot\cos(2\pi z) + a\cdot\sin(2\pi x) \\ B_Z = a\cdot\cos(2\pi x) + b\cdot\sin(2\pi y) + B_0 \end{cases}$$

The (x,y,z) is expanded into Fourier series in terms of (ξ,η,ζ). Fig.1 shows the GMC mesh of at equal intervals on the z = 0 plane in the Cartesian coordinates. The Poincaré map of magnetic surfaces is also overlapped in Fig.1. Here $\zeta = z$.

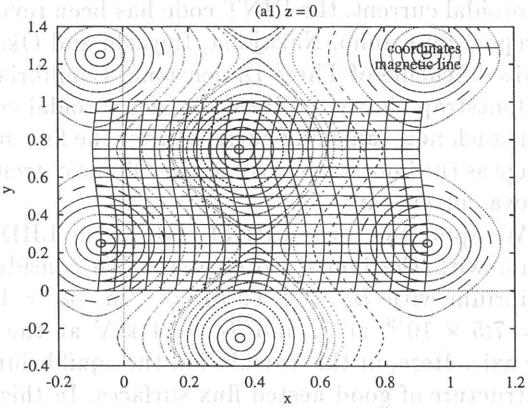

Fig.1. Poincaré map and GMC mesh in (x,y,z).

The deformation of the coordinate mesh follows that of the magnetic surface. For example, O-point locates at the same point of (ξ,η) regardless of ζ. The coordinates are well approximated by using Fourier expansion. Fig.2 shows the Poincaré map of magnetic surfaces in the GMC. In the GMC, the Poincaré map does not depend on ζ and it is plotted at the same point of (ξ,η) regardless of ζ. The improvement of method is required in order to treat general toroidal magnetic field.

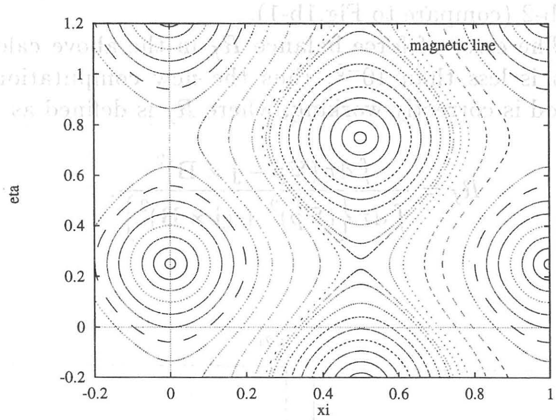

Fig.2. Poincaré map in the GMC.

Reference

[1]M.Kurata and J.Todoroki: J. Plasma Fusion Res. SERIES, Vol. **1** (1998) 491-494.