§30. Construction of Generalized Magnetic Coordinates by B-Spline Expansion

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The GMC are curvilinear coordinates (ξ,η,ζ), in which the magnetic field is expressed as

$$\mathbf{B} = \nabla\Psi(\xi,\eta,\zeta)\times\nabla\zeta + H^{\zeta}(\xi,\eta)\nabla\xi\times\nabla\eta,$$

where $H^{\zeta} = \sqrt{g}B^{\zeta}$ does not depend on ζ, and $\sqrt{g}$ is Jacobian. When the good magnetic surface exists, Ψ becomes independent of ζ and $\Psi(\xi,\eta) = Const.$ expresses the magnetic surface. When Ψ depends on ζ, $\overline{\Psi}(\xi,\eta)$ is the averaged magnetic surface which is obtained by averaging Ψ with respect to ζ. The breaking of magnetic surfaces like magnetic islands is investigated by using $\tilde{\Psi} = \Psi - \overline{\Psi}$. The GMC are to be constructed so that Ψ depends on ζ as little as possible. The function $\overline{\Psi}$ also corresponds to the averaged poloidal flux in the GMC.

The GMC was applied by using Fourier expansion in three dimensions to ABC magnetic field that can express topological toroidal magnetic field.[1,2] The coordinates are expanded to the same model field in Fourier series in the toroidal direction and the cubic B-spline function in other two dimensions in order to treat an aperiodic field and more complicated chaotic or ergodic region,

$$x = \xi + \sum_{l,m=1}^{M+3}\sum_{n=-N}^{N} \xi_{l,m,n}B_l(\xi)B_m(\eta)\exp(2\pi in\zeta),$$

$$y = \eta + \sum_{l,m=1}^{M+3}\sum_{n=-N}^{N} \eta_{l,m,n}B_l(\xi)B_m(\eta)\exp(2\pi in\zeta),$$

$$z = \zeta.$$

The coordinates are well constructed, but are influenced in the boundary by the boundary condition in the B-spline expansion. The quantity $\tilde{\Psi}$ is estimated as

$$E_{\xi\eta} = \oint\left\{\left|\tilde{H}^{\xi}\right|^2 + \left|\tilde{H}^{\eta}\right|^2\right\}d\zeta,$$

where $H^{\xi} = \sqrt{g}B^{\xi}, H^{\eta} = \sqrt{g}B^{\eta}$ and the tilde denotes ζ-dependent parts of them. The parameters are chosen such that $M = 80, N = 10$ and the magnetic field parameter $B_0 = 1.0$. Fig.1 shows the bird's-eye view of $E_{\xi\eta}$ in the GMC. The similar shape as the cases of $M = 40, 60$ appears with approximate quantity in case of $M = 80$, but it has the minute structure more than cases of $M = 40, 60$.

In the GMC rotational transform ι is expressed simply as

$$\iota = \frac{d\Psi_P}{d\Psi_T} = \left[\oint_C \frac{\overline{H}^{\zeta}(\xi,\eta)}{\left|\nabla\overline{\Psi}(\xi,\eta)\right|}dl\right]^{-1},$$

where Ψ_P, Ψ_T are poloidal and toroidal flux, the integral path C is taken on the averaged magnetic surface $\overline{\Psi}$. Fig.2 shows the comparison of the rotational transform calculated in the GMC and that obtained by field line tracing in the Cartesian coordinates. The two values are in good agreement.

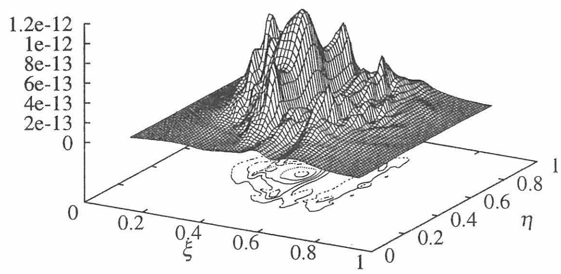

Fig.1. Bird's-eye View of $E_{\xi\eta}$ in the GMC.

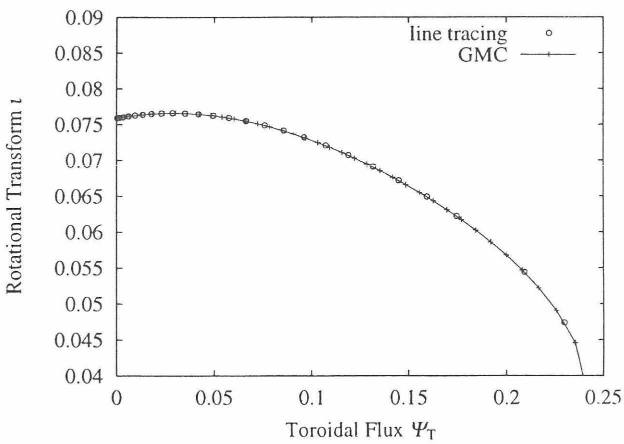

Fig.2. Rotational Transform.

Reference

1) Kurata, M. and Todoroki, J. : J. Plasma Fusion Res. SERIES, Vol. **1** (1998) 491-494.

2) Kurata, M. and Todoroki, J. : NIFS-PROC-40 (1999) 9-18.