§21. Kinematic Dynamos with Anisotropic α-Effect

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The combined action of rotation and buoyancy generally imparts helicity to turbulent motions. If the fluid is electrically conducting a mean electromotive force (or emf) results that is proportional to the components of the prevailing mean magnetic field, which is called "the alpha–effect". In the simplest (isotropic) case, the emf is parallel to the mean field, and this convenient ansatz is the one most commonly adopted. Nevertheless, an isotropic $\alpha-$effect is unrealistic, since the rotation significantly affects the motion of the fluid. This has led a number of authors to examine dynamos with an anisotropic $\alpha-$effect. Therefore, we studied electromagnetic induction by an $\alpha-$effect transverse to the rotation axis and by an $\alpha-$effect parallel to it.

We have derived a general formula for treating the α^2- and $\alpha\omega-$dynamos when the $\alpha-$effect is either transverse or longitudinal with respect to the rotational axis. The formulation follows the standard procedure of Bullard and Gellman (1954), so that its numerical calculation can be carried out using a program quite similar to the ones used in the calculation of ordinary α^2- and $\alpha\omega-$dynamos [1] with only a slight modification for the $\alpha-$effect.

We applied this method to the model of Busse and Miin (1979) and that of Rüdiger (1980), but extended the range of anisotropy from the originally studied transverse regime to the full interval of completely longitudinal–isotropic–completely transverse. The results are in good agreement with the ones of the original authors except where the anisotropy parameter x is more that 0.5 (Busse and Miin model) or near 1 (Rüdiger model). The bifurcation at the critical Reynolds number was always direct; we did not find complex eigenvalues in the entire range of anisotropy. Our numerical results were supported by the almost exact agreement of the solutions with those of the corresponding adjoint system [1].

Results of calculations with various parameters (anisotropy x and Reynolds number R_α) show some systematics in these dynamos [2]. When the anisotropy is essentially transverse ($x > 0.5$), the toroidal flux is expelled from the equatorial region toward the polar region, but it seems that the field structure is almost unchanged in other anisotropy range ($-1 < x < 0.5$). The correlation functions of the magnetic field with respect to the homogeneous dynamo ($x = 0$) shows that the only change occurs in the partition of magnetic energy between the dipole and quadrupole family harmonics. As the poloidal field is exclusively produced by the transverse $\alpha-$effect, this result can be understood in terms of the relative strength of the toroidal field generation compared with the poloidal field generation.

It is speculated that the dynamo action in the present models is largely determined by $(A_\alpha S_\beta T_\gamma)_L$ and $(A_\alpha T_\beta S_\gamma)_T$ interactions, where the subscripts indicate purely longitudinal or transverse $\alpha-$effect. The reason is that the ratio of the toroidal and poloidal field generation only depends on the anisotropy parameter x if we can neglect the $(A_\alpha S_\beta T_\gamma)_T$ interaction. The latter interaction only becomes important when x is large and $(A_\alpha T_\beta S_\gamma)_T$ becomes very small. This interaction tends to expel the toroidal flux from the equatorial region to high latitudes.

References

1) Kono, M. and P.H. Roberts, Geophys. Astrophys. Fluid Dyn. 67 (1992) 65.
2) Kono, M. and P.H. Roberts, Geophys. Astrophys. Fluid Dyn. 77 (1994) 27.