§27. The Role of Nonlinear Dynamics in Self-organization

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We have studied the role of nonlinear dynamics in the formation of global structures in plasma systems.

First, we studied a model of hole formation in Vlasov systems. Several steady state solutions have been proposed and they are presented in Schamel's review article[1]. However, they seem very artificial and complicated. Saeki and Genma proposed the simplest and physically interesting solution [2]: The essential point is to introduce particles with negative energy, namely trapped particles. The water-bag model is now written for the distribution function, which depends on phase variables only through the energy H.

$$f(H) = \begin{cases} \sqrt{\dfrac{m}{E_0}}\dfrac{n_0}{2\sqrt{2}} & (-e\phi_0 \le H \le E_0) \\ 0 & (\text{otherwise}) \end{cases} \tag{1}$$

The Saggeev potential for the potential defined by $\dfrac{d^2\phi}{dx^2} = -\dfrac{dV}{d\phi}$ is analytically given and implies that there is a maximum of ϕ, which corresponds to an electron hole.

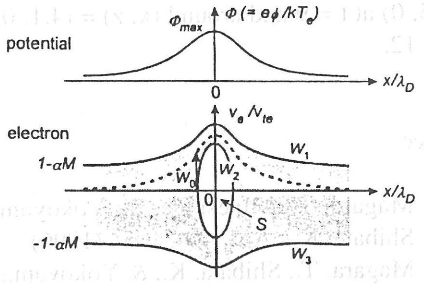

Fig.1. Spatial structure of the eletrostatic potential and phase space distribution of electrons

Dawson's one-dimensional plasma model[3] can be transformed into the system of hard spheres in a one-dimensional chain, each of which is under the harmonic potential around an equidistant equilibrium site. Namely if we denote by $x_i(t)$ and $v_i(t)$ the displacement from the equilibrium position and the velocity of the i-th sheet, then they form a harmonic oscillator $\dot{x}_i = v_i$ and $\dot{v}_i = -\omega^2 x_i$ as long as they do not collide. When a neighboring pair of sheets meet, namely, $x_i - x_{i+1} = a$, where a is the distance between the neighboring equilibrium sites, they exchange the momenta. Namely if one denotes the velocities before and after the collision by $v_i^<$ and $v_i^>$ respectively, we have $v_i^> = v_{i+1}^<$ and $v_{i+1}^> = v_i^<$. This collision process is usually introduced as an initial condition for the motion after the collision. However, we can now describe this hard sphere collision as a term in an autonomous differential equation[4]. The idea is to introduce a "half"-delta function $\delta_-(x)$, which is infinite only when $x = 0^-$. Then we have

$$\delta_-(x_i - x_{i+1} - a) = \frac{\delta_-(t - t_c)}{v_i^< - v_{i+1}^<} \tag{2}$$

Namely the delta function of time is concentrated just before the collision at time t_c. Thus the exchange of the velocity is expressed as

$$\dot{v}_i = -(v_i - v_{i+1})^2 \delta_-(x_i - x_{i+1} - a) \tag{3}$$

Indeed if we integrate both sides from $t = t_c + 0^-$ to $t = t_c + 0^+$, we have $v_i^> = v_{i+1}^<$.

Although Eq.(3) is singular in that it includes a delta function, it is autonomous and allows the orbital stability analysis, which is now in progress.

References
1) Schamel, H., Phys. Rep. **140** 161(1986)
2) Saeki, K, and Genma, H., Phys. Rev. Lett. **80** 1224(1998)
3) Dawson, J., Phys. Fluids **5** 445(1962)
4) Kitahara, K., unpublished(1998)