§23. A Periodic Motion Embedded in Plane Couette Turbulence

Kida, S.,
Kawahara, G. (Dept. Mech. Eng. Ehime Univ.)

A flow between two parallel plates which move with a constant velocity in opposite directions becomes turbulent at the Reynolds number above some critical value if it starts with a strongly disturbed state. This is called the plane Couette turbulence, the fluid motion in which is chaotic and never repeated. Nevertheless, it is known that the regeneration cycle is present to sustain near-wall coherent structures such as streamwise vortices and low-velocity streaks though its theoretical description has not been established. Here we report a periodic motion, discovered by solving the Navier–Stokes equation iteratively, which describes a full cycle of repetition of a series of dynamical processes including the formation and breakdown of coherent structures. Since it is unstable, this periodic motion is not attained in reality. However, the turbulent state spends most of the time around it. As a result, the mean velocity profile as well as the root-mean-squares of velocity fluctuations of the Couette turbulence coincide remarkably well with the temporal averages of the corresponding quantities of the periodic motion.

We solve the incompressible Navier–Stokes equations numerically by using a spectral method for a Couette turbulence. The Fourier expansions are employed in the streamwise and spanwise directions, and the Chebyshev-polynomial expansion in the wall-normal direction. Numerical computations are carried out on 8,448 grid points at Reynolds number $Re \equiv Uh/\nu = 400$, where U stands for half the difference of the two wall velocities, h is half the wall separation, and ν is the kinematic viscosity of fluid. The streamwise and spanwise computational periods are $L_x = 5.513h$ and $L_z = 3.770h$, respectively.

In the present numerical scheme the dependent variables are 31 for the mean streamwise and spanwise components of velocity, 7,424 for the wall-normal velocity and 7,936 for the wall-normal vorticity. The resulting number N of degrees of freedom of the present dynamical system is therefore 15,422. An instantaneous state of the flow field and its temporal evolution should be represented respectively, in principle, as a point and its trajectory in the N-dimensional phase space spanned by all the independent variables. In Fig. 1, we plot, with a grey line, a projection of the orbit over a period of $10,000h/U$ on the two-dimensional subspace spanned by the total energy input rate I and dissipation rate D which are normalized by those for a laminar state. Green dots are attached at every $2h/U$ time unit. The orbit generally tends to turn clockwise. The variation of the orbit, which is confined in a finite domain, is far from periodic. On the contrary, the frequency spectrum of the total kinetic energy is continuous, which suggests that it may be in a chaotic state.

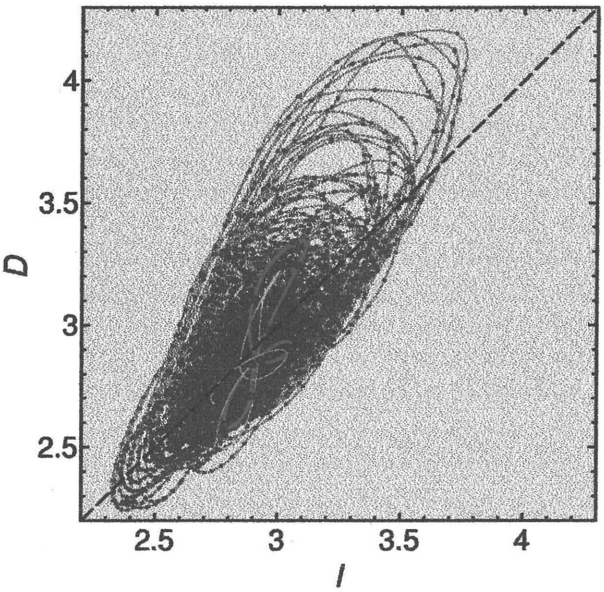

Fig. 1. Two-dimensional projections of a turbulent and periodic orbit.

Motivated by previous works on findings of the existence of periodic orbits embedded in a strange attractor in simple dynamical systems and on observations of the repetition of a series of dynamical processes in the present system, we searched a periodic orbit which might be embedded in the turbulent flow, and found one. A periodic orbit thus obtained is drawn with a closed red line, the period of which is $64.7h/U$. Green dots on the turbulence trajectory crowd much densely near the periodic orbit, implying that the turbulent state often approaches the periodic orbit. An example of such close approach is shown with yellow line which is a cut of the turbulence orbit. The turbulent state approaches the periodic orbit and follow it very closely for a while. The approach is, however, not permanent. The turbulence trajectory is destined to go away from the periodic orbit, sooner or later. In other words, this periodic orbit should be of saddle nature. This intermittent escape may be connected with the well-known bursting process which activates small-scale motions to enhance the energy dissipation. The present extraction of a periodic orbit may offer the first (to our knowledge) direct demonstration of the real existence of a periodic motion embedded in a turbulent flow.