§16. Model to Analyze Two-Dimensional Structures Including Poloidal Shocks and Geodesic Acoustic Modes in Toroidal Plasmas

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A variety of structures are formed in the toroidal plasmas, and their formation mechanism is one of the keys to understand the transport phenomena. In the edge transport barrier, a large poloidal flow with the poloidal Mach number M_p (poloidal flow velocity normalized by the poloidal sound velocity) ~ 1 is generated, so the poloidal shock [1)] can be formed. The poloidal shock structure is a steady density or potential jump in the poloidal direction, resulting from the plasma compressibility and the inhomogeneity of the magnetic field by the toroidicity, and is important, because it induces radial particle fluxes to accelerate the density pedestal formation on the L/H transition [2)]. Therefore, a model applicable both to the subsonic and sonic regimes is necessary to describe the formation process of the edge transport barrier during L/H transitions. The aim of this research is to present the comprehensive model for both of the regimes.

Extensions of the previous 2-D model [2)] are carried out to investigate two effects on the structural formation, i.e., the deviation from the Boltzmann relation and the parallel flow dynamics. The distribution of the potential and density different from the Boltzmann relation contributes to induce a particle pinch [3)], and the parallel flow dynamics is important to obtain the flow pattern, which contributes to the structural formation. These extensions enable to analyze the geodesic acoustic mode (GAM) [4)], which is the oscillatory zonal flow, caused by compressibility of the E × B flow in the presence of the geodesic magnetic curvature. Both the poloidal shock and the geodesic acoustic mode induce density asymmetry in the magnetic flux surface, so their competition must be examined to deepen our understanding of the transport barrier physics.

For analyzing the potential, density and flow velocity, a set of fluid equations consists of the momentum conservation equation, the continuity equation of the density, the charge conservation equation, and the Ohm's law. The main object in this research is the poloidally-asymmetric structure in the edge transport barrier, where a large poloidal flow is generated, and poloidal shocks can be formed. Here, the shock ordering [1)], which is the perturbations to be $O(\varepsilon^{1/2})$, is adopted, where ε is the inverse aspect ration. In the case in which $M_p \sim 1$, the steep structure in the poloidal direction is formed, and the perturbations become larger than $O(\varepsilon)$.

We assume density $n = n_i = n_e$, $B_\phi >> B_p$, low-β and electrostatic perturbations in a large aspect ratio tokamak with a circular cross-section. The set of equations for obtaining χ, M_{p0}, M_{p1} is derived:

$$\frac{\partial \chi}{\partial \tau} = -M_{p0}\varepsilon \sin\theta - \frac{\partial M_{p1}}{\partial \theta},$$

$$\frac{\partial M_{p0}}{\partial \tau} = \frac{B_0^{\,2} r}{m_i B_{p0}^{\,3} v_{ti}^{\,2} C_r^{\,2}} \left(\left\langle \frac{J B_p B_\phi}{n} \right\rangle - \left\langle \frac{\vec{B}_p \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_i}{n} \right\rangle_{\mathrm{bulk}} - \left\langle \frac{\vec{B}_p \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_i}{n} \right\rangle_{\mathrm{shear}} \right) + \frac{B_0^{\,2}}{B_{p0}^{\,2}} \varepsilon \left\langle \cos\theta \frac{\partial \chi}{\partial \theta} \right\rangle,$$

$$\begin{aligned} \frac{\partial M_{p1}}{\partial \tau} = {} & \hat{\mu}\;\; r^2 \frac{B_0}{B_{p0}} \frac{\partial^2}{\partial r^2} \left\{ M_{p0} \left[\exp(-\chi) - 1 \right] \right\} \\ & - \frac{2}{3} D \exp(-\chi) \frac{\partial^2 \chi}{\partial \theta^2} - \left(1 - M_{p0}^{\,2}\right) \frac{\partial \chi}{\partial \theta} \\ & - 2A \frac{\partial \chi^2}{\partial \theta} - \frac{\partial}{\partial \theta} \left[(1 - 2\chi) M_{p0} M_{p1} + \frac{1}{2} M_{p1}^{\,2} \right] \\ & + \varepsilon \left\{ D - \hat{\mu} \frac{B_0}{B_{p0}} \left[2r^2 \frac{\partial^2 M_{p0}}{\partial r^2} + 4r \frac{\partial M_{p0}}{\partial r} - 2M_{p0} \right] \right\} \cos\theta \\ & - 2\varepsilon M_{p0}^{\,2} \sin\theta, \end{aligned}$$

where $\chi(r,\theta) = \ln(n/\bar{n})$, $M_p \equiv \left(nB_0V_p\right)/\left(C_r \bar{n} B_p v_{ti}\right) = M_{p0}(r) + M_{p1}(r,\theta)$, $\tau = t/t_p$, $t_p = (B_0 r)/B_{p0} v_{ti} C_r$, $v_{ti} = \sqrt{2T_i/m_i}$, $C_r^{\,2} = 5/6 + T_e/(2T_i)$, $D = \left(4\sqrt{\pi} I_{ps} K_0 B_0\right)/\left(3\bar{n} v_{ti} C_r^{\,2}\right)$, $A = M_{p0}^{\,2}/2 + 5/\left(36 C_r^{\,2}\right)$, $\hat{\mu}$ is the shear viscosity coefficient, and the form of I_{ps} depends on M_p and the collision frequency [1)]. Here, the variables are divided into the average part and the perturbation part, which are denoted by subscript 0 and 1, respectively, and $V_r / V_p << 1$ is assumed, which is satisfied, even if a strong poloidal shock exists. Equations are obtained, keeping the terms up to $O(\varepsilon^{1/2})$ (shock ordering) and $O(\varepsilon)$ with the perturbation of each variable to be $O(\varepsilon)$ (small perturbation). This is because poloidal shocks and GAMs give $O(\varepsilon^{1/2})$ and $O(\varepsilon)$ perturbation, respectively, so the mediate parameter region must be considered to study their competition. Current J is obtained from Ohm's law, which includes polarization current $\varepsilon_0 \varepsilon_\perp \partial E_r/\partial t$, and external components as driven by an electrode, orbit losses, etc., where ε_0 is the vacuum susceptibility, and $\varepsilon_\perp$ is the perpendicular dielectric constant of a toroidal plasma.

This extended model includes the both generation mechanism of the poloidal shock structure and the GAM. The similar set of equations by previous studies, describing the poloidal shock structure or GAM, can be obtained from some limiting cases of our model. The derivation is the first step to analyze the multi-dimensionality of transport in the toroidal plasmas, which gives quantitative understandings of the transport barrier physics.

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