§26. Monte Carlo Simulation Code for Solving Radial Fluid Equations

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We develop a new Monte Carlo simulation code for overcoming difficulty in conventional Monte Carlo methods, where the difficulty is caused by nonlinear terms in fluid equations. To confirm the computational principle of the new method, we solve Dirichlet problems in one dimensional (1D, i.e., radial) coordinate space in the first trial. The code is called DIPS-1D (DIrichlet Problem Solver in 1D coordinate space).

In general, the fluid equation expressed in the form of the Fokker-Planck type equation can be rewritten as the following initial-boundary value problem (t is replaced by $t_1 - t$):

$$(L + \eta_*)u + \frac{\partial u}{\partial t} = h_*(t, \boldsymbol{x}) \ \text{ in } \mathcal{Q}, \tag{1}$$

$$u(t_1, \boldsymbol{x}) = \Phi(\boldsymbol{x}) \ \text{ on } \mathcal{M}, \tag{2}$$

$$u(t, \boldsymbol{x}) = G(t, \boldsymbol{x}) \ \text{ on } \mathcal{S}, \tag{3}$$

where $\mathcal{M}$ is a bounded domain with the boundary $\partial\mathcal{M}$, $\mathcal{Q} = \mathcal{M} \times [0, t_1)$, $\mathcal{S} = \partial\mathcal{M} \times [0, t_1)$, and

$$Lu = \left\{ \frac{1}{2} D^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + U_*^i \frac{\partial}{\partial x^i} \right\} u, \tag{4}$$

$$U_*^i = -U^i + \frac{1}{2} \frac{\partial D^{ij}}{\partial x^j}, \tag{5}$$

$$\eta_* = -\eta - \frac{\partial U^i}{\partial x^i}, \tag{6}$$

$$h_* = -h. \tag{7}$$

If Φ, G, h, η, D and $\boldsymbol{U}$ are assumed to be given-smooth-functions, the solutions of Eqs. (1)-(3) are known to be described as

$$\begin{aligned} &u(t, \boldsymbol{x}\,; \Phi, G, h_*, \eta_*, \mathsf{D}, \boldsymbol{U}_*) \\ &= E_{t,\boldsymbol{x}} \left[\Phi(\boldsymbol{X}(t_1)) \exp\left\{ \int_t^\tau \eta_*(s, \boldsymbol{X}(s)) \mathrm{d}s \right\} \chi_{\tau = t_1} \right] \\ &+ E_{t,\boldsymbol{x}} \left[G(\tau, \boldsymbol{X}(\tau)) \exp\left\{ \int_t^\tau \eta_*(s, \boldsymbol{X}(s)) \mathrm{d}s \right\} \chi_{\tau < t_1} \right] \\ &- E_{t,\boldsymbol{x}} \left[\int_t^\tau h_*(s, \boldsymbol{X}(s)) \exp\left\{ \int_t^s \eta_*(\vartheta, \boldsymbol{X}(\vartheta)) \mathrm{d}\vartheta \right\} \mathrm{d}s \right], \end{aligned} \tag{8}$$

where $E_{t,\boldsymbol{x}}$ is the expectation operator given by the diffusion process $\boldsymbol{X}(s)$ in coordinate space:

$$\mathrm{d}X^i(s) = \sigma_j^i(t, \boldsymbol{X}(s)) \mathrm{d}W^j(s) + U_*^i(t, \boldsymbol{X}(s)) \mathrm{d}s \tag{9}$$

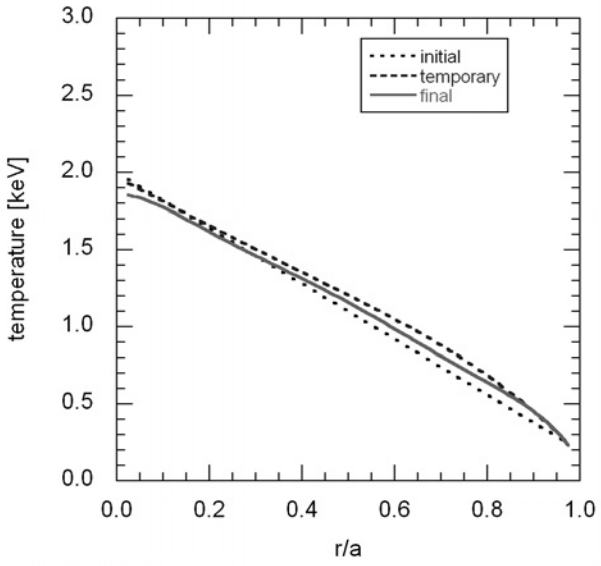

Fig. 1: Radial energy transport for electrons. Dotted line represents the initial guess $\Phi(r)$. Dashed line represents the solution found by DIPS-1D at the temporary step, and solid line represents the solution at the final step.

satisfying $\boldsymbol{X}(t) = \boldsymbol{x}$. Here $D^{ij} = \sigma_k^i \mathrm{g}^{k\ell} \sigma_\ell^j$, $\mathrm{g}^{k\ell}$ is the metric, $\boldsymbol{W}(s)$ is a Brownian process, χ_A is the indicator function of a set A (e.g., $\chi_{\tau<t_1} = 1$ if $\tau < t_1$, and $\chi_{\tau<t_1} = 0$ otherwise), and τ is defined as

$$\tau = \begin{cases} \text{the first time } \vartheta \in [t, t_1) \text{ that } \boldsymbol{X}(\vartheta) \text{ leaves } \mathcal{M} \\ \qquad\qquad\qquad\qquad \text{if such a time exists,} \\ t_1 \ \text{ otherwise.} \end{cases}$$

Using the Monte Carlo simulation code DIPS-1D based on Eq. (8), we solve iteratively the following radial energy transport equation for electrons.

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n T_\mathrm{e}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r \kappa_\mathrm{e}^{gB} \frac{\partial T_\mathrm{e}}{\partial r}\right) - \frac{3}{2}\langle \sigma v \rangle_\mathrm{rre} n^2 T_\mathrm{e} + S = 0, \tag{10}$$

where the initial condition is given as $\Phi(r) = T_\mathrm{ax}^{(0)} - \left\{ T_\mathrm{ax}^{(0)} - T_\mathrm{edge}^{(0)} \right\} r/a$ with $T_\mathrm{ax}^{(0)} = 2$ keV and $T_\mathrm{edge}^{(0)} = 200$ eV. The boundary conditions are $G(t, r_0) = T_\mathrm{e}(t, r_0) = T_\mathrm{e}(t, r_0 + \delta r) - \delta r\, \partial T_\mathrm{e}/\partial r(t, r_0)$ and $G(t, a) = T_\mathrm{e}(t, a) = 200$ eV with sufficiently small δr and r_0 (i.e., $0 < \delta r/a, r_0/a \ll 1$), and $\partial T_\mathrm{e}/\partial r(t, r_0) = -1.8$ keV/m. T_e is the electron temperature, and the constant density is assumed to be $n = 1 \times 10^{19}$ m^{-3}. κ_e^{gB} is the gyro-Bohm thermal conductivity, $a = 1$ m is the minor radius, $\langle \sigma v \rangle_\mathrm{rre}$ is the radiative recombination rate coefficient, and S is the heat source. The steady-state solution of Eq. (10) is given in Fig. 1. Details are shown in Ref. [1].

1) R. Kanno, S. Satake, M. Nunami, Plasma Fusion Res. (2011), in press.