§13. Path Integral Approach for Electron Transport in Peripheral Region

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From the viewpoint of development of stochastic methods, we consider a statistical analysis of an electron transport in the peripheral region of three dimensional toroidal plasma. If statistical property of perturbed field lines is not clear and effects of the Coulomb collisions can be represented as the Gaussian white noise, a motion of electron strongly tied to a field line is expressed as the following stochastic differential equations in the Cartesian coordinates (x, y, t) [1, 2];

$$\frac{dx}{dt} = \frac{B_x(x,y,t)}{B_t(x,y,t)} + W_x(t), \quad (1)$$

$$\frac{dy}{dt} = \frac{B_y(x,y,t)}{B_t(x,y,t)} + W_y(t), \quad (2)$$

where $\boldsymbol{B} = (B_x, B_y, B_t)$ is a magnetic field with $B_t > 0$, W_x and W_y are the Gaussian white noises with diffusion coefficients D_i. The Langevin equations (1) and (2) are simple as compared with the exact equations of motion which includes not only effects of the Coulomb collisions but also effects of neutrals, etc.; but effects of perturbed field lines can be easily investigated and the simple equation is a good candidate for fulfilment of the aim to develop basic techniques. Of course, the methods developed here are applicable to the exact equations, if the assumption of the Gaussian white noise describing effects of the collisions is appropriate.

A probability density is given by a path integral approach instead of solving directly the Langevin equations. The probability density $p(x_b, y_b, t_b)$ is written as

$$p(x_b, y_b, t_b) = \frac{1}{C} \lim_{\substack{N\to\infty \\ N_p\to\infty}} \frac{1}{N_p} \sum_{\ell=1}^{N_p} \prod_{i=x,y} e^{-\frac{(r_{i,b}-r_{i,a})^2}{4D_i(t_b-t_a)}}$$
$$\times \exp\left\{ \frac{\varepsilon}{2D_i} \sum_{j=0}^{N-1} \left[h_{i,j}^{(\ell)} \frac{r_{i,j+1}^{(\ell)} - r_{i,j}^{(\ell)}}{\varepsilon} - \frac{1}{2}\left(h_{i,j}^{(\ell)}\right)^2 \right] \right\}, \quad (3)$$

where $(h_x, h_y) = (B_x/B_t, B_y/B_t)$ means effects of field lines, N_p the number of Wiener paths $\{\boldsymbol{r}^{(\ell)}(t)\}$ starting from (x_a, y_a) and arriving at (x_b, y_b), $r_{i,k}^{(\ell)}$ the k-th element of the ℓ-th Wiener path $r_i^{(\ell)}(t)$, and C a normalizing factor. The elements of an Wiener path, $r_{i,1}, r_{i,2}, \ldots, r_{i,N-1}$, can be given by repeated application of the so-called 'interpolation formula': for $i = x, y$ and $j = 1, 2, \ldots, N-1$

$$r_{i,j} = \frac{(N-j)r_{i,j-1} + r_{i,N}}{N-j+1} + \xi_{i,j}\sigma_i \sqrt{\frac{(N-j)\varepsilon}{N-j+1}}, \quad (4)$$

where $r_{i,j} = r_i(t_j)$, and $\xi_{i,j} = \xi_i(t_j)$ means a Gaussianly distributed random variable ξ_i at $t = t_j$.

We demonstrate a Monte Carlo calculation of the electron distribution under both effects of the chaotic field lines and the collisions. According to methods of Eqs. (3) and (4), the Wiener paths $(x^{(\ell)}(t), y^{(\ell)}(t))$ are generated randomly and the weights on the paths are integrated. In actual calculation of an electron distribution in a disturbed magnetic field, we choose a magnetic field modeled as the Arnold-Beltrami-Childress (ABC) field:

$$\boldsymbol{B} = \begin{pmatrix} 0 \\ 0 \\ B_{t0} \end{pmatrix} + \begin{pmatrix} a\sin\omega t + c\cos\omega y \\ b\sin\omega x + a\cos\omega t \\ c\sin\omega y + b\cos\omega x \end{pmatrix}, \quad (5)$$

where a, b and c are parameters of the ABC field, and B_{t0} is constant satisfying $B_t > 0$. Evolution of distribution is shown as solid lines in Fig. 1. When the collisionality is large enough, the distribution for the x direction has a tendency to be the Gaussian-like distribution with the hollow profile and the diffusion coefficients of the collisions, and the one for the y direction is not seriously affected by effects of field lines.

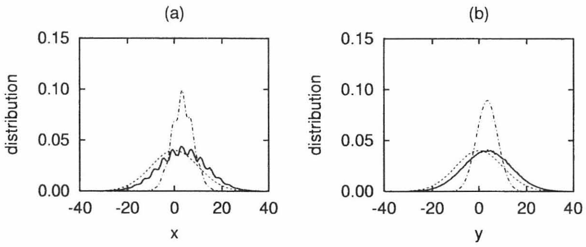

Figure 1: Evolution of distributions with $a = 15$, $b = 1$, $c = 1/2$, $\omega = \pi/2$, and $B_{t0} = 3$ for (a) the x direction and (b) the y direction. A dash-dot line is the distribution at $t_b = 1$ and a solid line is one at $t_b = 5$, where $D_x = D_y = 10$. A dot line is the Gaussian distribution with mean 0 and variance 100.

References

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