§22. Electron Thermal Conductivity in the Edge Region of Stellarators

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In the report by Kanno, Nakajima and Okamoto "Diffusion Phenomena in a Self-Similar Structure with Islands", we have derived the mean square displacement of a random particle in a fractal medium, which is given by

$$\left\langle (\Delta r)^2 \right\rangle = 2D\hat{\lambda}^{\beta/\alpha}, \tag{1}$$

where $\hat{\lambda} = \lambda/R$ and λ is the collisional mean free path in the z direction, and the diffusion coefficient in real space-time, D, is described as

$$D = \frac{2^{(1+\alpha^2-\beta)/\alpha}\pi^{2-\beta/\alpha}\Gamma(3/2\alpha)}{\Gamma(1/2\alpha)} \times R^2 \sum_{m,n} \left| \frac{\delta B_{m,n}(r)}{B_z} \right|^2 \delta_{nq(r)-m}. \tag{2}$$

When $\alpha = \beta = 1$, the diffusion coefficient D becomes one given by the quasilinear formula in Ref.[1]. Since the radial spreading of test electrons is the non-Gaussian process with fractal nature, the radial thermal conductivity for the collisionless limit will be given by

$$\chi_r = \frac{\left\langle (\Delta r)^2 \right\rangle}{2\tau} = \frac{D}{\tau}\hat{\lambda}^{\beta/\alpha}, \tag{3}$$

where $\tau = \lambda/v_t$ is the time interval and v_t is the electron thermal velocity.

Dimensions of fractal space-time, α and β, are defined from the self-similar structure with islands. We can consider that because of the self-similarity of the structure, the cross section of the cylinder is a kind of the Sierpinski gasket (SG) or carpet (SC) [2]. Thus, dimensions of space and time are defined as

$$\alpha = \frac{\ln N}{\ln \delta}, \tag{4}$$

$$\beta = \frac{\alpha \ln(\delta^2 - 1)}{\ln \delta^2}, \tag{5}$$

where $N = N_0/\delta$; N_0 is the increase-rate of the number of areas except for islands, e.g. $N_0 = \delta^2 - 1$ for the SG/SC, and $\delta = d_0/d$. Here, d_0 and d are widths of the whole fractal structure and the largest island, respectively.

As shown in eq.(3), we have the new formula for the electron thermal conductivity in a self-similar structure with islands for the collisionless limit. If the size of islands is small ($\delta \gg 1$), then $\alpha \sim 1$ and $\beta \sim 1$, i.e. the diffusion becomes the Brownian given in Ref.[1]. While, if the size is large enough ($+\infty \gg \delta > 1$), the thermal conductivity is reduced as compared with one in the only stochastic field, because $\chi_r \propto \hat{\lambda}^{\mu}$ and $\mu = \beta/\alpha = \ln(\delta^2 - 1)/\ln \delta^2 < 1$. We can interpret this result as follows. Since the random motion of electrons is suppressed by fractal islands and an electron has to take a detour around islands, the diffusion becomes slow effectively. While, if islands are not large and do not make the fractal structure, the effect of islands on the diffusion is negligible, because the dimension of the structure with islands projected onto the r or the θ axis is equal to unity, i.e. $\alpha = \beta = 1$.

Using eq.(3), we estimate the radial thermal conductivity of electrons, χ_r. Considering the thermal conductivity in the Large Helical Device (LHD) [3] with the major radius $R = 3.9$ m, and assuming that 1) the island structure in the edge region of LHD is represented by Sierpinski's model and 2) the edge temperature $T_e^{\mathrm{edge}} = 1$ keV and the edge density $n_e^{\mathrm{edge}} = 10^{18}$ or 10^{17} m^{-3} are realized by the high temperature divertor plasma operation, we obtain $\hat{\lambda}/2\pi = \lambda/2\pi R \approx 5 \times 10^2$ or 5×10^3, and the radial thermal conductivity χ_r shown in Fig.1. In Fig.1, the conductivity χ_r is normalized by the Rechester and Rosenbluth conductivity χ_r^{st}, which is defined as [1]

$$\chi_r^{\mathrm{st}} = \frac{\pi R^2 \hat{\lambda}}{\tau} \sum_{m,n} \left| \frac{\delta B_{m,n}(r)}{B_z} \right|^2 \delta_{nq(r)-m}. \tag{6}$$

Thus, we can see that χ_r is reduced for a case of $\delta < 10$. Note that in the Sierpinski model, δ is bounded larger than two. [2]

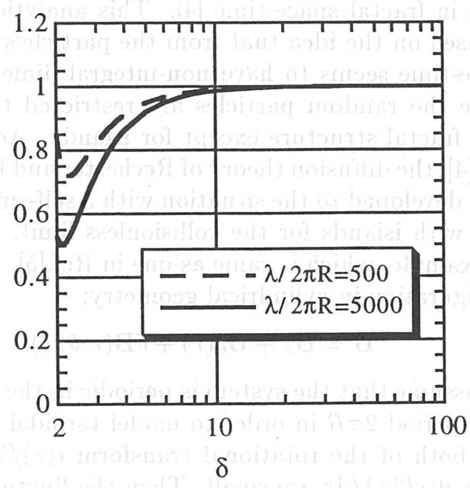

Fig. 1. The radial thermal conductivity of electrons.

References

[1] Rechester,A.B. and Rosenbluth,M.N. : Phys. Rev. Lett. **40** (1978) 38.

[2] Mandelbrot,B.B. : *The Fractal Geometry of Nature* (W.H.Freeman, New York, 1982) Chap.14.

[3] Iiyoshi,A. et al. : Fusion Technol. **17** (1990) 169.