§30. Analysis on Time Evolution of Shielding Current Density in High-Temperature Superconductor

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I. Introduction

Since the evaluation of the shielding current density is indispensable for the design of engineering applications of high-temperature superconductors (HTSs), several numerical methods [1), 2)] have been so far proposed to calculate the shielding current density. After discretization with respect to space, the governing equation of the shielding current density is reduced to a system of ordinary differential equations (ODEs). If implicit schemes have been applied to the system, the nonlinear equations have to be solved at each time step. However, the equations are extremely time-consuming. Therefore, the system of ODEs should be solved with the method other than implicit schemes.

The purpose of this study is to develop a fast method for analyzing the shielding current density in an HTS film.

II. Governing Equations

An HTS film of thickness b is exposed to the time-dependent magnetic field $\boldsymbol{B}/\mu_0$ and it has a square cross section Ω. Under the thin-plate approximation, the shielding current density $\boldsymbol{j}$ can be expressed as

$$\boldsymbol{j} = \nabla \times [(2S/b)\boldsymbol{e}_z] \ ,$$

and its time evolution is governed by the following equation [2)]:

$$\mu_0 \frac{\partial}{\partial t}\left[\hat{Q} + \frac{2}{b}\hat{I}\right]S = -\frac{\partial}{\partial t}\langle \boldsymbol{B}\cdot\boldsymbol{e}_z\rangle - (\nabla\times\boldsymbol{E})\cdot\boldsymbol{e}_z \ . \tag{1}$$

Here, $\boldsymbol{E}$ denotes an electric field and $\langle\ \rangle$ represents an average operator through the thickness. In addition, $\hat{I}$ denotes an identity operator and $\hat{Q}$ is defined by

$$\hat{Q}S = \iint_\Omega Q(|\boldsymbol{x}-\boldsymbol{x}'|)S(\boldsymbol{x}',t)d^2\boldsymbol{x}' \ .$$

Here, the integration kernel $Q(r)$ is given as

$$Q(r) \equiv -(\pi b^2)^{-1}[r^{-1} - (r^2+b^2)^{-1/2}] \ .$$

As the J-E constitutive relation, the following power law is assumed:

$$\boldsymbol{E} = E(|\boldsymbol{j}|)(\boldsymbol{j}/|\boldsymbol{j}|) \ , \tag{2a}$$

$$E(j) = (j/j_C)^N \ , \tag{2b}$$

where E_C and j_C denote a critical electric field and a critical current density, respectively, and N is a constant.

The initial and the boundary conditions to (1) are assumed as follows: $S = 0$ at $t = 0$ and $S = 0$ on $\partial\Omega$. Here, $\partial\Omega$ denotes the boundary of Ω.

III. High-Speed Method

After the spatial discretization by means of the FEM, the initial-boundary-value problem of (1) is reduced to a nonlinear system of ODEs. However, the system cannot be always solved with the Runge-Kutta method even when an adaptive step-size control (ASSC) algorithm [3)] is incorporated.

For the purpose of suppressing an overflow in the ASSC, we propose that the power law (2) be modified as follows:

$$\boldsymbol{E} = E^*(|\boldsymbol{j}|)(\boldsymbol{j}/|\boldsymbol{j}|) \ , \tag{3a}$$

$$E^*(j) = (j/j_C)^{N^*(j/j_C)} \ , \tag{3b}$$

where

$$N^*(z) \equiv (N-2)\phi(8[z-(z_U+z_L)/2]/(z_U-z_L)) + 2 \ .$$

Here, z_U and z_L are both constants and $\phi(x) = (1-\tanh x)/2$. Note that $E(j) \cong E^*(j)$ is fulfilled for $j/j_C \le z_L$. Hence, if the solution of (1) and (3) satisfies $|\boldsymbol{j}|/j_C \le z_L$, it also becomes an approximate solution of (1) and (2). For this reason, we propose the following method for analyzing the time-evolution of the shielding current density:

1) After assuming (3) as the J-E constitutive relation, the initial-boundary-value problem of (1) is solved by means of the Runge-Kutta method with an ASSC.
2) Whether the resulting solution satisfies or not is checked numerically. If the solution fulfills the inequality, it is acceptable as a solution of (1) and (2).

In order to compare the speed of the proposed method with that of the conventional method, the CPU times required for both methods are measured (see Fig. 1). The proposed method is much faster than the backward Euler method. From this result, we can conclude that the proposed method becomes a powerful tool for the analysis of the shielding current density.

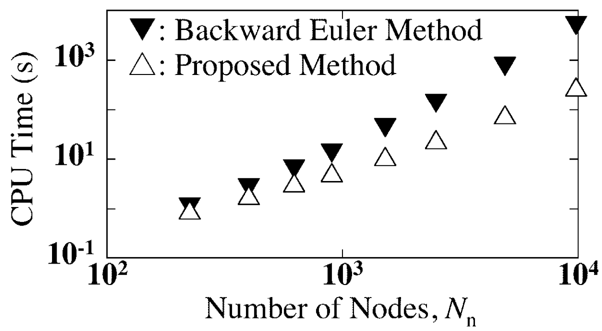

Fig. 1. Dependence of the CPU times on the number N_n of nodes for the case with $N = 16$, $E_C = 1$ mV/m and $j_C = 1$ MA/cm^2.

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2) Kamitani, A., Takayama, T., Tanaka, A., Ikuno, S.: Physica C, to be published.
3) Press, W. H., Teukolsky, S. A., Vetterling, W. T., Flannery, B. P.: Numerical Recipes in Fortran 77, 2nd ed., New York: Cambridge University Press, 1992, Chap. 16.