§ 23. Circulation in a Short Cylindrical Couette System

Kageyama, A. (Earth Simulator Center, JAMSTEC), Ji, H. (PPPL, Princeton Univ.), Goodman, J. (Princeton University Observatory)

Laboratory experiments using liquid metal have made important tests of magnetohydrodynamic (MHD) theories and processes. Examples include laboratory demonstration of the Alfvén wave, magnetofluid convection, and dynamo action. Recently, a new kind of liquid-metal MHD experiment, motivated by astrophysics, has been proposed by the authors[1, 2] to study magnetorotational instability (MRI), which is believed to dominate the transport of angular momentum in electrically conducting accretion disks. Liquid gallium will be used in a Couette flow between cylinders of radii $r_1 < r_2$ and angular velocities $0 < \Omega_2 < \Omega_1$ but $r_2^2\Omega_2 > r_1^2\Omega_1$ so that the flow will be stable against conventional Taylor-Couette instabilities (TCI).

In previous linear stability analyses of gallium Couette flow[1, 2], we adopted periodic boundary condition in the vertical (axial) direction, ignoring the effects of the top and bottom interior surfaces of the vessel ("end-caps"). The choice of vertical boundary conditions is probably inconsequential when the height of the flow (H) is much larger than the gap width, as in Taylor's classic experiments[3]. Our experimental volume $\pi H(r_2^2 - r_1^2)$ will be limited by the availability of gallium, a far more expensive fluid than water, while the gap must be wide enough so that the magnetic diffusion time is not much shorter than the rotation period. These considerations drive us to an aspect ratio $H/(r_2 - r_1) \sim O(1)$, in which the endcaps may assume great importance.

We have performed a water experiment and complementary numerical simulations to study the effects of the endcaps and, if possible, to find a way to set up a short Couette flow that is unstable to MRI yet stable against TCI. Since the viscosities of the two fluids are similar, standard visualization techniques in water serve to predict the flow structure in the opaque liquid gallium, at least in the absence of magnetic field.

The flow is very different from that of an ideal, infinitely long Couette system. Simulations show that endcaps corotating with the outer cylinder drive a strong

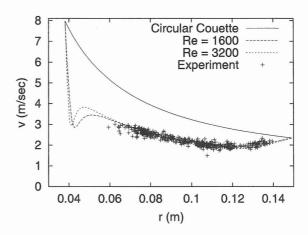


Figure 1: Comparison of azimuthal flow profiles obtained by simulation and experiment.

poloidal circulation that redistributes angular momentum. Predicted toroidal flow profiles agree well with experimental measurements (See Fig. 1).

Further detailed numerical studies reveal a strong radially inward flow near both end-caps. After turning vertically along the inner cylinder, these flows converge at the midplane and depart the boundary in a radial jet. To minimize this circulation in the MRI experiment, end-caps consisting of multiple, differentially rotating rings are proposed. Simulations predict that an adequate approximation to the ideal Couette profile can be obtained with a few rings.

Details of this study will be reported in [4].

References

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