

§23. Self-sustained Turbulence and H-mode Confinement in Toroidal Plasmas

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The equation for the dressed test mode for the nonlinear ballooning mode turbulence in the presence of the radial electric field has been derived [1] as

$$d^2 p / d\eta^2 + (\alpha\Lambda / K)\{1 - (1/2 + \alpha - s)\eta^2\} p - M\Lambda[1 + 3(s - \alpha)^2\eta^2] p + L_1 p = 0$$

$$L_1 p \equiv \omega_{E1} [KF^{-1} d^3 p / d\eta^3 - \Lambda(1 + M/K)F^2 dp / d\eta]$$

In this equation, normalization is:  $r / a \rightarrow \hat{r}$ ,  
 $t / \tau_{Ap} \rightarrow \hat{t}$ ,  $\chi \tau_{Ap} / a^2 \rightarrow \hat{\chi}$ ,  $\mu \tau_{Ap} / a^2 \rightarrow \hat{\mu}$ ,  
 $\tau_{Ap} / \mu_0 \sigma_c a^2 \rightarrow 1 / \hat{\sigma}$ ,  $\lambda \tau_{Ap} / \mu_0 a^4 \rightarrow \hat{\lambda}$ ,  $\gamma \tau_{Ap} \rightarrow \hat{\gamma}$ .  
We employ the notation  $\tau_{Ap} \equiv a \sqrt{\mu_0 m_i n_i} / B_p$ ,  
 $\Lambda = \hat{\lambda} n^4 q^4$ ,  $K = \hat{\chi} n^2 q^2$ ,  $M = \hat{\mu} n^2 q^2$ ,  
 $F = 1 + (s\eta - \alpha \sin \eta)^2$ ,  $s = r (dq / dr) / q$ ,  
 $\omega_{E1} = \tau_{Ap} (dE_r / d\hat{r}) (srB)^{-1}$  and  $\alpha = -q^2 R \beta'$ .  
Other notation is standard.

The eigenfunction is written as

$$p(y) = u_0 + \sum_{n=1} a_n u_n$$

where the function  $u_n$  is explicitly given as  
 $u_n = H_n(y) \exp(-y^2/4)$  and  $H_n(y)$  is the n-th order Hermite function. The odd and even parity modes are mixed by the operator  $L_1$  when  $\omega_{E1} \neq 0$ . The normalized eigenvalue is calculated by

$$\Omega = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n} \langle 0 | L_1 | n \rangle \langle n | L_1 | 0 \rangle.$$

(In the previous work [1], only neighbouring terms were kept.) The turbulent transport coefficient is given as

$$\hat{\chi}_H = \frac{\hat{\chi}_L}{1 + G_1 \omega_{E1}^2}$$

$$G_1 = 8 \alpha (1 + 2\alpha - 2s) \left\{ 2 + \frac{6(s - \alpha)^2}{1 + 2(\alpha - s)} \right\}^{-1}$$

$$\times \left[ \frac{3}{8} (1 - (s - \alpha)^2) + \frac{2(1 + 6(s - \alpha)^2)}{4\alpha} \right]^2$$

$$+ \frac{1}{2} \left[ \frac{1 - 5(s - \alpha)^2}{4} - \frac{2(s - \alpha)^2}{\alpha} \right]^2 + \frac{3}{8}$$

The contour lines of  $\hat{\chi}_H$  are illustrated in Fig.1 on the  $\alpha - \omega_{E1}$  plane. The correction appears in the numerical coefficient  $G_1$ .

The cross-field momentum flux can be calculated by the relation

$$P_{\theta,r} = -\mu_H \nabla V_{\theta}.$$

Normalized momentum flux is given as  $\hat{P}_{\theta,r} = \hat{\mu}_H \omega_{E1}$ , and the normalized velocity gradient is expressed by  $\omega_{E1}$ . Figure 2 illustrates  $\hat{P}_{\theta,r}$  as a function of  $\omega_{E1}$ . Viscosity is calculated and is close to  $\hat{\chi}_H$ . It is shown that the radial momentum flux is a decreasing function of the velocity shear in the high rotation shear limit. This indicates that, for a fixed momentum flux, two solutions of the rotation shear are available. This provides a basis for the bifurcation in the radial electric field structure in the H-mode modelling.

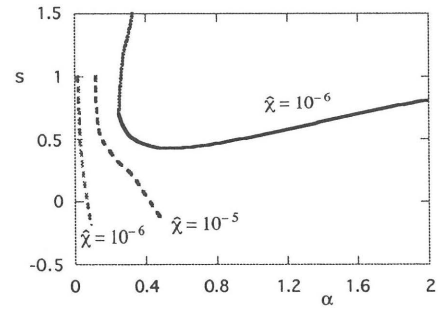


Fig.1 Contour plot of the normalized thermal conductivity as a function of  $(\alpha, \omega_{E1})$ .

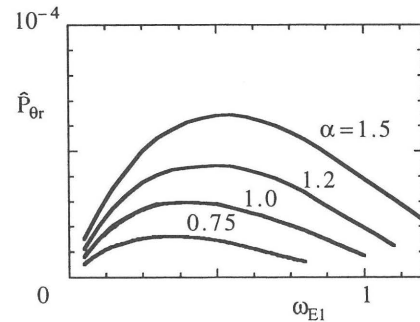


Fig.2 Anomalous cross-field momentum flux  $\hat{P}_{\theta,r}$  as a function of the velocity gradient  $\omega_{E1}$ .

Reference

- [1] Itoh S-I, Itoh K, Fukuyama A and Yagi M  
1994 *Phys. Rev. Lett.* **72** 1200