§34. Turbulent Transport Coefficient in High Rayleigh Number Regime

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Turbulent transport coefficients, viscosity and heat diffusivity, are examined for the fluid where the buoyancy works. Two dimensional fluid which has the temperature gradient against the gravity, g, is treated. High Rayleigh number regime, $R_a >> R_{ac}$, is considered. The method of dressed test mode is employed for an inclusion of the nonlinearlity due to the Lagrange derivatives. The back-ground turbulence is renormalized into the form of diffusivity on a test mode through two mode interactions. A recurrent formula of the nonlinear interactions in a form of diffusivity is obtained. The nonlinear marginal stability condition for the test mode gives the relation between the force acting on the fluid and the effective dissipation rates. For the fluid bounded by the stiff wall, the boundary fluid is also effective in determining the turbulence level: The Nusselt number is in proportion to $R_a^{1/3}$.

We consider the fluid which has a temperature difference between the boundaries $z = 0$ and $z = d$. The equations of the fluid motion and heat transport are given as

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = -\frac{\nabla p}{\rho} + g + v_c \Delta v \qquad (1)$$

$$\frac{\partial T}{\partial t} + v \cdot \nabla T = \kappa_c \Delta T \qquad (2)$$

where p, ρ, v_c and κ_c are the pressure, mass density, molecular viscosity and molecular thermal conductivity.

By use of the renormalization, the nonlinear marginal stability condition is obtained as [1]

$$R_a^t = \frac{k_\perp^2}{k_x^2} k_\perp^4 d^4 \qquad (3)$$

where the turbulent Rayleigh number is defined as

$$R_a^t = \frac{\alpha g d^4 |\nabla T_0|}{(v_T + v_c)(\kappa_T + \kappa_c)} \qquad (4)$$

where v_T and κ_T are the turbulent viscosity and thermal conductivity, respectively. The fluctuation level is expressed in terms of stream function as

$$v_T(v_T + v_c) = \kappa_T(\kappa_T + \kappa_c) = \Sigma \frac{|\phi_1|^2}{(2 + R_a^t k_x^2 k_\perp^{-6} d^{-4})} \qquad (5)$$

Equations (3)-(5) determines the turbulent transport coefficients, v_T and κ_T, and the fluctuation level as a function of the temperature gradient. As a result of the strong transport, the temperature profile can be modified. In the large R_a limit, the turbulent Prandtl Number approaches to unity, and the temperature gradient in the core is suppressed by the factor $(R_a / R_{ac})^{-1/9}$. Due to this change, the Nusselt number is in proportion to $R_a^{1/3}$.

In the conventional approach like eddy viscosity model [2], linear growth rate is simply balanced with nonlinear damping. This method consistently treats the nonlinear interaction and driving force, and allows further application to nonlinear instabilities.

1) S.-I. Itoh, et al., Research Report FURKU-95-07 (1995).
2) V. M. Canuto, et al., Phys. Fluids 30 (1987) 3391.