# §17. Periodic Change of Solar Differential Rotation

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The periodic oscillation of the inhomogeneous rotation of the sun is studied by use of the MHD dynamo theory [1]. There exists a turbulent electromotive force which is driven by the vorticity of the flow (i.e., the $\gamma$ -dynamo). In addition, its counterpart exists in the vorticity equation, that is, the rotation is induced by inhomogeneous magnetic field in turbulent plasma. Based on this dynamo theory, a periodic change of solar differential rotation with the period of 11 years is theoretically explained. The predicted amplitude is compared with observations.

The dynamo theory of turbulent plasma has provided the mean field equation as

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times \left( 2\left(\boldsymbol{u} - \frac{\gamma}{\beta}\boldsymbol{B}\right) \times \boldsymbol{\omega}_F + \nu_T \nabla^2 \left(\boldsymbol{u} - \frac{\gamma}{\beta}\boldsymbol{B}\right) \right) . \quad (1)$$

Equation (1) describes an interesting effect of the magnetic field on the *generation* of global vorticity. The terms which have the coefficient $\nu_T$ in the RHS of Eq. (1) are induced by the microscopic turbulence. The term $\nabla \times \nu_T \nabla^2 \boldsymbol{u}$ is the well-known turbulent viscosity term, i.e., the localized vorticity decays in time. It tends to eliminate the inhomogeneity of the vorticity. (For instance, the tachocline is weakened by this term.) On the contrary, the term $\nabla \times \nu_T \nabla^2 (\gamma \boldsymbol{B}/\beta)$ generates the velocity so that the velocity $\boldsymbol{u}$ becomes parallel to $\boldsymbol{B}$. If the localized magnetic field exists, then this dynamo term *generates* the localized flow profile.

From Eq. (1), we study that the response of the toroidal velocity appears against the change of the magnetic field $\boldsymbol{B}$. When the change is slow in comparison with the diffusion time, the response of the relative velocity $\boldsymbol{u}$ in the presence of the temporal variation of $\boldsymbol{B}$, $\delta \boldsymbol{u}$, is given as

$$\delta \boldsymbol{u} = \gamma \beta^{-1} \boldsymbol{B} . \quad (2)$$

When we apply this result (2) to the case of sun, the modulation of the toroidal velocity by the dynamo magnetic field is deduced. Here we use polar coordinates $(r, \zeta, \theta)$. The dynamo magnetic field is known to have a strong toroidal magnetic field. This magnetic field is localized in the mid- and low-latitude regions. Equation (2) shows that the azimuthal velocity in the rotation frame is stronger in the mid- and low-latitude regions. This localized azimuthal flow has an up-down symmetry. The polarity rule of solar dynamo is well known: the sign of dynamo magnetic field is opposite between northern and southern hemispheres. It should be also noticed that the dynamo coefficient $\gamma$ is a pseudoscalar while $\beta$ is a scalar. That is, the ratio $\gamma/\beta$ changes the sign in the northern and southern hemispheres. From these facts, the induced modification of the velocity $\delta \boldsymbol{u}$ has an up-down symmetry. This symmetry property of the induced velocity $\delta \boldsymbol{u}$, together with the localization in the latitude, is common to the profile of the solar rotation velocity.

We next estimate the magnitude of the induced velocity $\delta \boldsymbol{u}$. The strength of the magnetic field is evaluated as about 1T or less in the convective zone of the sun. The location $r/r_{SUN} \sim 0.8 - 0.9$ may be relevant as the representative value in the study of the inhomogeneous rotation velocity. As the mass density in this region [2], we adopt the number density of hydrogen as $O(10^{28})\ m^{-3}$ or $\rho \sim 10 \mathrm{kgm}^{-3}$, which gives that $B \sim 300$ m/s for $B = 1\mathrm{T}$ in the Alfven unit. An estimate of the ratio of $|\gamma/\beta| = O(10^{-2}) \sim O(10^{-1})$ has been given for the study of the $\gamma$ -dynamo mechanism for the generation of solar magnetic field [2]. If one employs the mean value of the range this estimate, $|\gamma/\beta| \simeq 10^{-1.5}$, we have an evaluation of $\delta u$ as

$$|\delta u| \sim 10 \text{ m/s} . \quad (3)$$

This is a few per cent of the differential rotation velocity in the solar convective zone.

The solar magnetic field shows a quasi-periodic change with the period of about 22 years. As a result of this periodic change of $\boldsymbol{B}$, the induced velocity $\delta \boldsymbol{u}$ is also subject to the (quasi-)periodic change. Two cases can be considered depending on the changeability of the sign of $\gamma/\beta$. If the sign of $\gamma/\beta$ is not altered by the change of the polarity of the magnetic field, then $\delta \boldsymbol{u}$ changes its direction and magnitude with the period of 22 years. In the opposite case, i.e., $\gamma/\beta$ changes the sign together with $\boldsymbol{B}$, then $\delta \boldsymbol{u}$ changes its magnitude with the period of 11 years. It is therefore plausible that the periodic oscillation of differential rotation is composed of the component with the 11-year period and the one with 22-year period.

These theoretical predictions are compared with observational results. First of all, the resemblance of the spatio-temporal patterns of the periodic change of rotation and of the magnetic activity is understood naturally. Second, the amplitude of the periodic oscillation is in a range of observation: the amplitude of oscillation of $1 - 5\ \mu\mathrm{Hz}$ has been observed in the upper convective zone. The radial profile of the amplitude of oscillation is reported, and $\delta \boldsymbol{u}$ is shown to have larger amplitude near the surface. The main elements of the observation on the periodic change can be explained by the $\gamma$ -dynamo theory.

**Reference**

[1] Itoh S-I, et al. 2005 Astrophys. J. **618** 1044
[2] Yoshizawa A, et al. 2000 Astrophys. J. **537** 1039