§35. Statistical Equation for Strong Plasma Turbulence

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The method of self-sustained turbulence [1] has allowed to analyze the strong turbulence. However, the random noise was not kept in the previous formalism, and the statistical description was not complete; The stationary state was given as a stable fixed point [1], and the method was limited to understand the statistical property of the strong plasma turbulence. In order to study the statistical nature, the formalism of Langevin equations is developed for a strong plasma turbulence.

The basic set of equations in [1] has the form

$$\frac{\partial}{\partial t} f + \mathcal{L}^{(0)} f = \mathcal{N}(f) + \tilde{S}_{th}, \tag{1}$$

where $\mathcal{L}^{(0)}$ denotes the linear operator, $f = (\phi, J, p)^T$ and $\mathcal{N}(f)$ stands for the nonlinear terms.

A Langevin equation is formulated from Eq.(1) based on the statistical physics consideration. In formulating a Langevin equation, the nonlinear term is separated into the effective damping term $\gamma_i f_i$ and the random noise term $\tilde{S}_i$. The process of separation and the relation with the method of dressed test mode are discussed by introducing a model projection operator. One k-Fourier component of eq.(1) is chosen as

$$\partial f_k/\partial t + \mathcal{L}^{(0)} f_k = \mathcal{N}_k = \sum_{k'} \mathcal{M}_{kk'} f_{k'} f_{k-k'} \tag{2}$$

where the suffix k is suppressed if not necessary. The projection operator $\mathcal{P}$ is introduced to filter, from $\mathcal{N}$, the component which is statistically dependent on f_k. The projected term $\mathcal{P}\mathcal{N}_k$ is correlated with f_k. The rest, $(1-\mathcal{P})\mathcal{N}$, is statistically independent of f_k, and is called an incoherent, or, random source.

A model projection is considered as follows. The fundamental assumption in the analysis is that the system has large number of positive Lyapunov exponents, and the excited fluctuations are approximately statistically independent of each other. One set of turbulent state, $\{f_{i,k'}\}$, is chosen and the process, that the test mode is taken away from this set, is considered. By the reduction of the test mode by the amount of $-\delta f_k$, the modification in the background fluctuations $\{-\delta f_{k''}\}$ appear. This is the induced variation of the background fluctuations associated with the change of test mode. This influenced variation, "polarization", is not statistically independent of f_k. With this change, the nonlinear term on the test mode is coherently modified by the amount of

$$\delta\mathcal{N}_k = \sum_{k'} \mathcal{M}_{k,-k'} f_{-k'} \left(\partial/\partial t + \mathcal{L}^{(0)} - \delta\mathcal{N}'/\delta f\right)^{-1}_{k+k'} \mathcal{M}_{k,k'} f_{k'} \delta f_k$$

From this relation, we formally pose a model projection operator $\mathcal{P}\mathcal{N}_k$ as

$$\mathcal{P}\mathcal{N}_k = \left\{\sum_{k'} \mathcal{M}_{k,-k'} f_{-k'} \left(\partial/\partial t + \mathcal{L}^{(0)} - \delta\mathcal{N}/\delta f\right)^{-1}_{k+k'} \mathcal{M}_{k,k'} f_{k'}\right\} f_k \tag{3}$$

This is approximated by the effective diffusion operator, $\mathcal{P}\mathcal{N}_{i,k} = -\mu_{i,k} k_\perp^2 f_{i,k}$, or $\mathcal{P}\mathcal{N}_{i,k} = \gamma_{i,k} f_{i,k}$.

The rest part is rewritten as

$$\tilde{S} = (1-\mathcal{P})\mathcal{N}(f) \tag{4}$$

and is considered to be an incoherent noise term. As noted before, the separation of collisional drag and noise in thermal fluctuations is given by the fluctuation-dissipation theorem. A separation of nonlinear interaction terms in turbulent fluctuations is undetermined and is self-consistently determined later. Then a Langevin equation is derived as

$$\frac{\partial}{\partial t} f + \mathcal{L} f = \tilde{S} + \tilde{S}_{th} \tag{5}$$

with

$$\mathcal{L}_{ij} = \mathcal{L}_{ij}^{(0)} + \gamma_i \delta_{ij} \tag{6}$$

(δ_{ij} is the Kronecker's delta) and $\tilde{S} = (\tilde{S}_1, \tilde{S}_2, \tilde{S}_3)$.

The operator to the k-th component, $\mathcal{L}_k$, is the renormalized operator, which includes the effective transfer rates expressed as

$$\gamma_{i,k} = -\sum_{\Delta} M_{i,kpq} M^*_{i,qkp} \theta^*_{qkp} \left|\tilde{f}^2_{1,p}\right| \tag{7}$$

The self-noise has a much shorter correlation time, and is approximated to be given by the Gaussian white noise term $w(t)$ as

$$\tilde{S}_{i,k} = w(t) \sum_{\Delta} M_{i,kpq} \sqrt{\theta_{kpq}}\, \zeta_{1,p} \zeta_{i,q} \tag{8}$$

The term $\zeta_{j,p}$ in a random noise represents the j-th field of q-component in the nonlinear term $\mathcal{N}$, and their correlation functions satisfy the average relations of the mode, which we call an Ansatz of equivalence in correlation in the following, as

$$\langle \zeta_i \zeta_j \rangle = \langle f_i f_j \rangle \text{ and } \langle \zeta_{i,p} \zeta_{j,q} \rangle \propto \delta_{pq} \tag{9}$$

This Langevin formulation of the nonlinear plasma instability allows us to study the statistical nature of the strong plasma turbulence.

References

[1] Itoh. K., et al.; Plasma Phys. Contr. Fusion **38** (1996) 2079.
[2] Itoh, S.-I. and Itoh, K.; Research Report IPP III/234 (Max-Planck-Inst. für Plasmaphysik, 1998)