§27. Impact of Turbulence Spreading on Subcritical Turbulence in Inhomogeneous Plasmas

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Recent progress of the plasma turbulence has shown that the fluctuations are possibly excited by nonlinear instabilities, i.e., through subcritical excitations, and that the spreading of turbulence has considerable impact on the turbulent transport. We study the influence of the turbulence spreading on the self-sustained turbulence of subcritical instabilities. It is shown that there is a minimum radial plasma extent, in order for a self-sustained turbulence to exist in linearly stable plasmas [1]. The new phase boundary in parameter space is derived, in which the effect of turbulence spreading is included.

We study a one-dimensional model to study the turbulence spreading into stable regions. The turbulence quantities are averaged over the magnetic surface, and a profile in the $x-$ direction is studied. We take the case that the plasma has a subcritical instability in the region of $0 \le |x| \le L$, and it is strongly stable in the regions $|x| > L$.

A dynamical equation is formulated in a form as

$$\frac{\partial}{\partial t} I = \Lambda I + \chi_0 \frac{\partial}{\partial x} I^{\alpha} \frac{\partial}{\partial x} I , \qquad (1)$$

where $I = |\tilde{\phi}^2| / |\tilde{\phi}^2_{\rm local}|$ is the normalized electrostatic potential fluctuation amplitude, $|\tilde{\phi}^2_{\rm local}|$ is the level which is given by the local balance of drive and damping $\Lambda = 0$, Λ is the decorrelation rate (including the growth rate and local nonlinear damping rate), and $\chi_0 I^{\alpha}$ is the diffusion coefficient due to a turbulence spreading, in which the dependence on the fluctuation level is represented by use of the index α. (In a weak turbulence limit, $\alpha = 1$ holds, and $\alpha = 1/2$ for strong turbulence limit.) This model has several limitations. First, the fluctuations, the wavelengths of which are comparable to L, are not included. Second, the incoherent and fluctuating kicks are not taken into account. Noticing that these additional processes may have a substantial influence, we choose the model (1) as a starting point of the analysis.

We introduce a characteristic length $\ell = \sqrt{\frac{\chi_0}{(1+\alpha)\Lambda_0}}$, and the lengths and Λ are normalized with respect to ℓ and Λ_0, respectively, $\zeta = x/\ell$, $\hat{L} = L/\ell$, $\hat{\Lambda} = \Lambda/\Lambda_0$ By use of this normalization, Eq.(1) is rewritten as

$$\frac{d^2}{d\zeta^2} F + \hat{\Lambda} F^{1/(1+\alpha)} = 0 , \qquad (2)$$

where a new variable for the fluctuation amplitude is introduced as $F = I^{1+\alpha}$.

We have the solution

$$\int_0^{F(0)} \frac{dF}{\sqrt{H(F(0)) - H(F)}} = \hat{L} , \qquad (3)$$

where the Sagdeev potential is given as

$$H(F) \equiv 2\int_0^F dF\, \hat{\Lambda}\, F^{1/(1+\alpha)} \qquad (4)$$

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The asymptotic relation for the turbulent transport coefficient, which has the dependence $\chi = \chi_0 I^{\alpha}$, is given as

$$\chi = \chi_0 \left\{ 1 - \frac{\alpha}{1+\alpha} \sqrt{\frac{1}{2|\Lambda'(1)|}} \exp\left(-\sqrt{|\Lambda'(1)|}\, \hat{L} \right) \right\} \qquad (5)$$

This analysis provides a generalized Maxwell's construction rule as

$$H(1) \ge \exp\left\{ -2\sqrt{|\Lambda'(1)|} \left(2\hat{L} - \int_0^{F_*} \frac{dF}{\sqrt{-H(F)}} \right) \right\} \qquad (6)$$

In the absence of turbulence spreading (i.e., $\hat{L} \to \infty$), this recovers standard result of Maxwell's constraction rule, $H(1) \ge 0$.

Reference:
[1] K. Itoh, S.-I. Itoh, T.S. Hahm, P.H. Diamond J. Phys. Soc. Jpn., Vol.**74**, No.7 (2005) 2001