§30. A Model of Sawtooth Based on the Transport Catastrophe

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The sawtooth oscillation in tokamaks is characterized by the repetitive and rapid crash of the central electron temperature. Observations on the TEXTOR tokamak concluded the fact that the q value at the magnetic axis remains below unity[1]. The temperature profile shows fast flattening, but the magnetic structure is changed only little by the crash. This finding stimulated the theory which could simultaneously treat the full temperature crash and partial change of the magnetic structure. One theoretical approach to study the rapid grow of the m=1 mode is to investigate the effect of current diffusivity [1]. The feature of the subcritical turbulence was studied in analyzing the sawtooth cycle [2]. Although the sawtooth physics is progressed, a variety in sawtooth oscillations has been noticed experimentally. In some cases, the m=1 mode may not be essential in causing the crash.

To resolve this, we have proposed a new model of the sawtooth which is independent of the growth of the m=1 mode. Recently, the theory of the anomalous transport and magnetic braiding has been developed by authors [3]. If the pressure gradient exceeds a certain threshold, the anomalous transport coefficient is subject to the catastrophic bifurcation. The hysteresis curve of the thermal conductivity was derived. This mechanism provides an understanding for the sawtooth which is not triggered by the burst of the m=1 helical mode.

The ratio $\zeta (=\chi_e/\chi_i)$ and the ion viscosity m are used as $\chi_i = \mu$ and $\chi_e = \zeta \mu$. In the electric turbulence limit, the relation $\zeta = 1$ holds. In the limit of magnetic turbulence with the complete braiding, we have $\zeta = \sqrt{m_i T_e/m_e T_i}$.

When the pressure gradient exceeds a threshold as,

 $G_0 > G_c \approx 0.5 \text{ s}$

where $G_{0e,i} = (R/a)\Omega'(Rd\beta_{0e,i}/dr)$, the magnetic braiding takes place,

$$\zeta \left(\equiv \frac{\chi_{e}}{\chi_{i}} \right) = 1 + \sqrt{\frac{m_{i}T_{e}}{m_{e}T_{i}}} \left(1 - \frac{\mu_{c}}{\mu} \right) \Theta(\mu - \mu_{c})$$

The dynamical equation is derived by taking into account the transport catastrophe.

The development of the transport coefficient

$$\begin{split} d\mu/dt &= \sqrt{G_0} \tau_{Ap}^{-1} (1 - \mu/\mu_s) \, \mu \\ \chi_{e \ s} &= \chi^L \left(\frac{G_{0i} + G_{0e}/\zeta}{G_{0i} + G_{0e}} \right)^{3/2} \zeta^2 \end{split}$$

and those for the ion pressure gradient and electron pressure gradient, as

$$\frac{\mathrm{d}G_{\mathrm{0i}}}{\mathrm{d}t} = \frac{1}{\tau_{\mathrm{Ec}}} \left(G_{\mathrm{L}\,i} - \frac{\mu}{\chi_{i}^{\mathrm{L}}} G_{\mathrm{0i}} \right)$$
$$\frac{\mathrm{d}G_{\mathrm{0e}}}{\mathrm{d}t} = \frac{1}{\tau_{\mathrm{Ec}}} \left(G_{\mathrm{Le}} - \frac{\mu\zeta}{\chi_{e}^{\mathrm{L}}} G_{\mathrm{0e}} \right)$$

This set of coupled equations are solve. The rapid decay at the critical pressure gradient, the sudden appearance and disappearance of the magnetic perturbation with the lower-mode numbers, and the start of the pile up after the end of the burst is recovered. The time scale is not characterized by the resistivity, so that the fast crash is possible. Figure 1 illustrates the details of the development at the crash.



Fig.1 Development of the pressure gradient, transport coefficient and magnetic perturbation.

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