§28. Noise-Excitation of Global Mode and Transient Transport Problems

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Fluctuations of the long-wave-length mode of the order of minor radius in background micro fluctuations are considered. According to the extended Fluctuation Dissipation Theorem, its fluctuation level and associated thermal flux have been given [1]. A formula of the heat flux is

$$q_{\ell} = -\frac{\sum_{\ell} \frac{k_{\ell,y}^2}{\gamma_{\ell,p} - \lambda_{\ell}} \frac{1}{2\lambda_{\ell}} |A_{\ell,11}|^2 \left\langle \tilde{\boldsymbol{S}}^* \tilde{\boldsymbol{S}} \right\rangle p'_0}{\left\langle \tilde{\boldsymbol{S}}^* \tilde{\boldsymbol{S}} \right\rangle p'_0}$$

where the suffix ℓ indicates the long-wave-length mode, λ_{ℓ} is the eigenvalue of the nonlinear decorrelation rate, A_{ℓ} is the projection operator, and \tilde{S} is the nonlinear noise for the long-wavelength mode, respectively. The over-bar denotes the integral over the correlation length of the ℓ mode. This level of Eq.(215) is the statistical average of the amplitude, and radial shape is given by eigenmode in toroidal plasmas. The nonlinear noise which is driven by the background microscopic turbulence is given in the limit of $|p| \ll |k|$ as

$$\langle \boldsymbol{S}^* \boldsymbol{S} \rangle \simeq \hat{C}_0^h \gamma_v^{h^2} p^2 k^{-6} \chi_{turb}$$

where *p* and *k* are the wave numbers of the longwave-length mode and micro mode, respectively, χ_{turb}

is the turbulent transport coefficient which is induced by the microscopic mode fluctuations. One has the heat flux which is induced by the long-wave-length fluctuations as

$$q_{\ell} \simeq - \overline{\sum_{\ell} \frac{\hat{C}_{0}^{h}}{k^{2}} \chi_{turb} p_{0}^{'}} \sim - \overline{p^{2} k^{-2} \chi_{turb} p_{0}^{'}} .$$

with the help of the relation $\lambda_{\ell} \simeq \chi_{turb} p^2$.

This result gives an insight into the transient transport problems. First, this heat flux is small in stationary state in comparison with the local heat flux which is driven by the microscopic turbulence. The latter is given as $-\chi_{turb} p'_0$. The nonlocal heat flux is about $p^{2k^{-2}}$ times smaller than that in the stationary state. However, the nonlocal heat flux is influential in the transient response. The heat flux changes over the distance of the correlation length, which is of the order of p^{-1} . When the noise source of the eigenmode of

the *p*-mode suddenly changes at a radius, the change of the global mode amplitude appears. The change of mode amplitude propagates across the plasma radius as a response of an radial eigenmode, and reaches the average in an autocorrelation time λ_{ℓ}^{-1} after a local change happens. That is, the statistical change of the heat flux due to the fluctuating global mode is realized in a time interval of λ_{ℓ}^{-1} . We see that the longwave-length mode induces, after a local impulse, a statistical change of heat flux, magnitude of which is $(p/k)^2(-\chi_{turb} p'_0)$, at a distance of p^{-1} in a time interval of λ_{ℓ}^{-1} .

This change of heat flux is not a diffusive process. Nevertheless, if one interprets this change as a diffusion process, then this change of heat flux may be attributed to an effective diffusivity χ_{eff} . When a transient delta-function perturbation $X_0\delta(x)$ is given at t=0, the diffusion process gives the perturbation δX , which has a form $\delta X \sim X_0 \frac{1}{\sqrt{\chi_{eff} t}} \exp\left(-\frac{x^2}{\chi_{eff} t}\right)$. The exponential dependence is dominant, and one has

 $-\ln(\delta X/X_0) \sim x^2/\chi_{eff}t$. Based on this relation, the observed relative change is interpreted by the effective diffusivity as

$$\chi_{eff} = \frac{(\text{distance})^2}{\text{time interval}} \left| \ln(\text{relative change}) \right|^{-1}$$

Let us estimate χ_{eff} for the nonlocal heat flux of concern. One has distance $\approx 1/p$, time interval $= \lambda_{\ell}^{-1}$ and relative change $\approx p^{2}k^{-2}$. The effective thermal diffusivity is estimated as $\chi_{eff} \approx p^{-2}\lambda_{\ell} \left| \ln (p/k)^{2} \right|^{-1}$. We have an effective diffusivity as

$$\chi_{eff} \simeq \left| \ln \left(p/k \right)^2 \right|^{-1} \chi_{turb} .$$

This value is modified by the factor $\left| \ln \left(p/k \right)^2 \right|^{-1}$

from χ_{turb} . This is much larger than the heat diffusivity in a stationary state, since

 $\ln(p/k)^2\Big|^{-1} >> p^2k^{-2}$ holds.

These analyses show that the contribution of the long-wave-length mode, which is excited by the statistical process of micro-turbulence, has strong influence in the response of energy transport after the transient perturbation.

Reference

1) Itoh S-I and Itoh K Plasma Phys. Contr. Fusion 43 (2001)