§28. Transport Coefficient Induced by Drift Wave Turbulence Screened by Zonal Flows

Itoh, K.,
Hallatschek, K. (IPP), Itoh, S.-I. (Kyusyu Univ.), Diamond, P.H. (UCSD), Toda, S.

In this work, an explicit forms of the transport coefficient and the Dimits shift is discussed for a practical use [1].

An analytic estimate for the quasi-linear driving coefficient of zonal flows by drift waves, D_{rr}, has been given

$$D_{rr} \simeq \frac{1}{B^2}\frac{k_\theta^2}{\gamma_L}\left|\tilde{\phi}\right|^2,$$

in the vicinity of the marginal condition of the linear stability $\Delta\omega_k \simeq \gamma_L$. It is given, in terms of the normalized fluctuation amplitude, as $D_{rr} \simeq \left(k_\theta^2 k_\perp^{-4}\right)\omega_*^2\gamma_L^{-1}\hat{\phi}^2$. The growth rate of the zonal flow energy has been introduced by the definition $2D_{rr}q_r^2 = \alpha\hat{\phi}^2$. That is, the time rate α is given as

$$\alpha \simeq \frac{\omega_*}{\gamma_L}\frac{2k_\theta^2 q_r^2}{k_\perp^4}\omega_*.$$

The Dimits shift is given by the critical condition, which is explained in terms of the linear growth rate,

$$\gamma_{L,c} = \frac{4(1-\mu)^2}{\mu^2 H^2}\frac{q_r^2}{k_\theta^2}\alpha.$$

(See [1] for the expression of the parameters H and μ.) Elimination α, at $\gamma_L = \gamma_{L,c}$, provides an equation of the critical growth rate $\gamma_{L,c}$ at the boundary of Dimitz shift as

$$\gamma_{L,c} = \frac{2\sqrt{2}(1-\mu)}{\mu H}\frac{q_r^2}{k_\perp^2}\omega_*.$$

For the least stable mode, q_r is estimated as, $q_r \simeq \frac{\sqrt{1-\mu}}{2}K_0$, which gives an estimate of $\gamma_{L,c}$

$$\gamma_{L,c} = \frac{(1-\mu)^2}{\sqrt{2}\,\mu H}\frac{K_0^2}{k_\perp^2}\omega_*.$$

One estimate for $K_0 = k_r$:

$$\gamma_{L,c} = \frac{(1-\mu)^2}{\sqrt{2}\,\mu H}\frac{k_r^2}{k_\perp^2}\omega_*$$

For parameters $\mu \simeq 1/2$, $\gamma_{L,c}$ is of the order of one-tenth of ω_*.

The fluctuation amplitude is given as follows.

(a) Small growth rate limit: In the case of weak instability, i.e.,

$$\gamma_L < \frac{1}{(1-\mu)}\frac{k_\perp^4}{k_\theta^2 q_r^2}\nu_{damp} \qquad \text{[region I]}$$

the fluctuation level is given by

$$\hat{\phi} = \frac{\gamma_L}{\omega_*} \equiv \hat{\phi}_I$$

(b) Intermediate growth rate limit: For the case of

$$\frac{1}{(1-\mu)}\frac{k_\perp^4}{k_\theta^2 q_r^2}\nu_{damp} < \gamma_L < \gamma_{L,c}, \qquad \text{[region II]}$$

the fluctuation level is given by

$$\hat{\phi} = \frac{1}{\sqrt{1-\mu}}\frac{k_\perp^2}{k_\theta q_r}\sqrt{\frac{\nu_{damp}}{\omega_*}\frac{\gamma_L}{\omega_*}} \equiv \hat{\phi}_{II}.$$

(c) Large growth rate limit: The transition from the collisional-damping-dominated region [region II] to the nonlinearity-dominated region is expected to occur at

$$\frac{1}{\mu H\rho_s^2 k_\theta^2}\nu_{damp} + \gamma_{L,c} < \gamma_L. \qquad \text{[region III]}$$

One has,

$$\frac{\Delta\omega_k}{\omega_*} \simeq \frac{\mu H\rho_s^2 k_\perp^4}{4(1-\mu)q_r^2}\left(-1+\sqrt{1+\frac{8(1-\mu)q_r^2}{\mu H\rho_s^2 k_\perp^4}\left(\frac{\gamma_L-\gamma_{L,c}}{\gamma_L}\right)}\right)\frac{\gamma_L}{\omega_*} \equiv \hat{\phi}_{III}$$

The asymptotically-linear dependence on γ_L in this model is recovered, and a suppression factor appears. The suppression factor, which is induced by the co-existence of the zonal flow, is approximately evaluated as $\sqrt{\mu H/2(1-\mu)}\,\rho_s k_\perp^2 q_r^{-1} \sim k_\perp\rho_s$.

A similar argument is possible for the thermal conductivity. In the regions I and II, a fitting formula is given as

$$\chi_{I+II} = \frac{\gamma_L\sqrt{\nu}}{\sqrt{\gamma_L}+\sqrt{\nu}}\frac{1}{k_r^2}.$$

Where $\nu = (1-\mu)^{-1}k_\perp^4 k_\theta^{-2}q_r^{-2}\nu_{damp}$ denotes the impact of collisional damping of the zonal flow. In the region III, one has

$$\chi_{III} = \frac{\mu H\rho_s^2 k_\perp^4}{4(1-\mu)q_r^2}\left(-1+\sqrt{1+\frac{8(1-\mu)q_r^2}{\mu H\rho_s^2 k_\perp^4}\left(\frac{\gamma_L-\gamma_{L,c}}{\gamma_L}\right)}\right)\frac{\gamma_L}{k_r^2}$$

.

A fitting in the regions I, II and III provides a formula of turbulent transport coefficient which is screened by zonal flows as

$$\chi_i = \chi_{fit} \equiv \sqrt{\chi_{I+II}^2 + \chi_{III}^2\Theta(\gamma_L-\gamma_{L,c})}$$

where $\Theta(\gamma_L-\gamma_{L,c})$ is a Heaviside function.

References

[1] Itoh, K., K. Hallatschek, S.-I. Itoh, P.H. Diamond and S. Toda, Phys. Plasmas Vol. **12** (2005) 062303