§26. Energy Partition between Fluctuations and Zonal Flows

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In this study, the higher-order corrections by zonal flow on the zonal flow drive are renormalized, and the driving term at an arbitrary magnitude of zonal flow vorticity is derived [1]. Based on the nonlinear form of the zonal flow growth rate, the steady state solution is obtained. In the collisionless limit, the turbulence level is shown to vanish while the zonal flow remains at finite amplitude, when instability is weak. The critical condition for the onset of drift wave turbulence in the presence of zonal flow is derived. The turbulent transport, including the zonal flow effects, is obtained. The partition ratio of fluctuating field energy among the drift wave turbulence and zonal flow is also obtained. A comparison with DNS [2] is also made.

The model dynamical system for the drift wave action N_k, $N_k = \left(1 + k_\perp^2 \rho_s^2\right)^2 |\tilde{\phi}_k|^2$, and the zonal flow velocity V_Z are introduced as

$$\frac{\partial}{\partial t} U = \frac{\partial^2}{\partial r^2} \frac{c^2}{B^2} \int d^2k \frac{k_\theta k_r}{\left(1 + k_\perp^2 \rho_s^2\right)^2} \hat{N}_k - \gamma_{damp} U ,$$

$$\frac{\partial}{\partial t} N_k + \frac{\partial \omega_k}{\partial \boldsymbol{k}} \cdot \frac{\partial N_k}{\partial \boldsymbol{x}} - \frac{\partial \omega_k}{\partial \boldsymbol{x}} \cdot \frac{\partial N_k}{\partial \boldsymbol{k}} = 0 ,$$

where U is the vorticity of the zonal flow $U = \partial V_Z / \partial r$. By expansion with respect to the vorticity of the zonal flow as, $\hat{N}_k = \hat{N}_k^{(1)} + \hat{N}_k^{(2)} + \hat{N}_k^{(3)} \cdots$, where $\hat{N}_k^{(n)}$ is the n-th order term of U

, the evolution equation for the zonal flow is rewritten as

$$\frac{\partial}{\partial t} U = \sum_m^\infty G^{(m)} - \gamma_{damp} U \quad .$$

By use of the higher order kinetics for the drift wave packets in the zonal flow field, we have

$$\frac{\partial}{\partial t} U = \frac{q_r^2 D_{rr}}{1 + \dfrac{H k_\theta^2 \rho_s^2 U^2}{\Delta\omega_k^2 + \Gamma^2}} U - \left(\mu_{\parallel}\left(1 + 2q^2\right) q_r^2 + \nu_{\rm damp}\right) U$$

where $D_{rr} = -\dfrac{c^2}{B^2} \displaystyle\int d^2k \frac{R(q_r, \Omega) k_\theta^2 k_r}{\left(1 + k_\perp^2 \rho_s^2\right)^2} \frac{\partial N_k}{\partial k_r}$ is the result of the quasilinear theory, q_r is the wavenumber of zonal flow, $\Delta\omega_k$ is the decorrelation rate of drift waves, $\Gamma = \max\left(\omega_b, q_r v_{g\,r}\right)$, ω_b is the bouncing frequency od drift wave packet, $\nu_{\rm damp} \simeq \nu_{ii}/\varepsilon$ is the collisional damping rate, H is a numerical coefficient, and $\mu_{\parallel}$ is the turbulent shear viscosity for the flow along the field line and q is the safety factor.

This result gives the partition of energy between the microscopic fluctuations and zonal flows. The other limit of interest is the collisionless limit, $\nu_{\rm damp}/q_r^2 D_{rr} \to 0$. In this case, the fluctuation amplitude $\tilde{\phi}$ vanishes at a critical vorticity of zonal flow,

$$U = U_c ,$$

where

$$U_c = \max\left(\frac{2(1-\mu)}{\mu H}, \sqrt{\frac{2(1-\mu)}{\mu H}} k_r \rho_s\right) q_r V_{\rm d} ,$$

and $V_{\rm d}$ is the diamagnetic velocity. (See [1] for details.) The analytic theory in this wrok and the result of direct numerical simulation is compared in Fig.1. Satisfactory agreement is obtained.

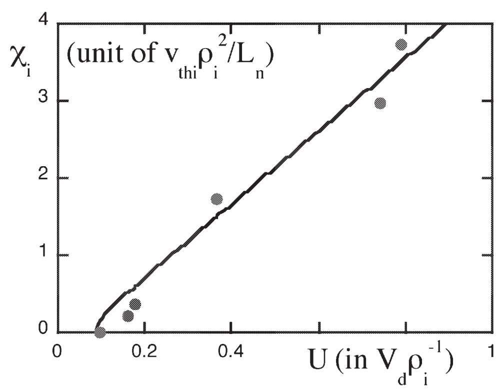

Fig.1 Comparison of the relations $\chi_i(U)$ for the steady state of ITG mode. Zonal flow vorticity is measured in a unit of $V_{\rm d} \rho_i^{-1}$ and thermal conductivity is in $v_{\rm thi} \rho_i^2 L_n^{-1}$. Theory (solid line) and DNS data (dots) quoted from [2].

References

[1] Itoh, K., K. Hallatschek, S.-I. Itoh, P.H. Diamond and S. Toda,Phys. Plasmas Vol. **12** (2005) 062303
[2] K. Hallatschek: Phys. Rev. Lett. **93** (2004) 065001.